An Integrated Component of Your Study Manual Program

Actuarial University
Your Path to Success

Study Manual

GOAL
Practice. Quiz. Test. PASS

Virtual eFlashcards

Videos & More!

a/s/m

Exam FAM-L Study Manual

1st Edition, 3rd Printing

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

a/s/m
Actuarial Study Materials
Learning Made Easier

a/s/m

Exam FAM-L Study Manual

1st Edition, 3rd Printing

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

TO OUR READERS:

Please check A.S.M.'s web site at **www.studymanuals.com** for errata and updates. If you have any comments or reports of errata, please e-mail us at **mail@studymanuals.com**.

Welcome to Actuarial University

Actuarial University is a reimagined platform built around a more simplified way to study. It combines all the products you use to study into one interactive learning center.

You can find integrated topics using this network icon.

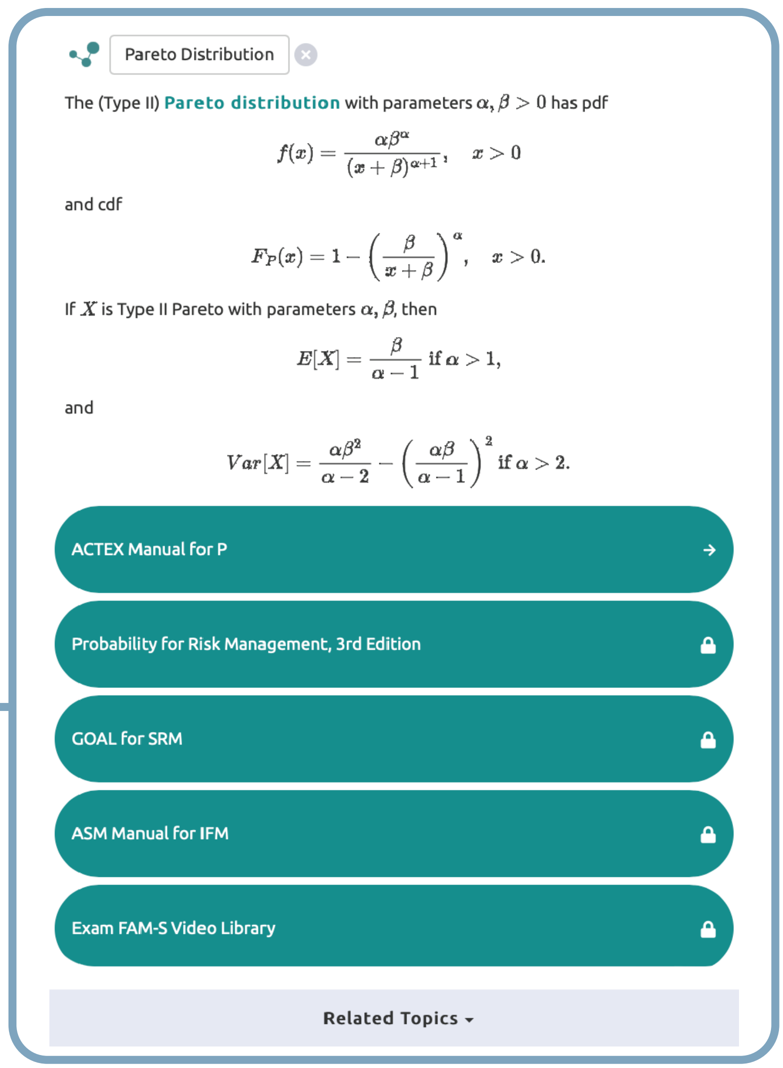
When this icon appears, it will be next to an important topic in the manual. Click the **link** in your digital manual, or search the underlined topic in your print manual.

1. Login to: www.actuarialuniversity.com

2. Locate the **Topic Search** on your exam dashboard and enter the word or phrase into the search field, selecting the best match.

3. A topic "**Hub**" will display a list of integrated products that offer more ways to study the material.

4. Here is an example of the topic **Pareto Distribution:**



Within the **Hub** there will be unlocked and locked products.

Unlocked Products are the products that you own.

ACTEX Manual for P →

Locked Products are products that you do not own, and are available for purchase.

Probability for Risk Management, 3rd Edition

Many of Actuarial University's features are already unlocked with your study program, including:

Instructional Videos*	**Planner**
Topic Search	**Formula & Review Sheet**

Make your study session more efficient with our Planner!

Planner

Template ACTEX FM Study Manual - New 2022 syllabus

Begin Study 07/01/2023 **End Study** 11/14/2023

✔	7/1/2023 - 7/16/2023	Interest Rates and the Time Value of Money
✔	7/16/2023 - 8/12/2023	Annuities
✔	8/12/2023 - 8/27/2023	Loan Repayment
✔	8/27/2023 - 9/15/2023	Bonds
✔	9/15/2023 - 9/22/2023	Yield Rate of an Investment
✔	9/22/2023 - 10/11/2023	The Term Structure of Interest Rates
✔	10/11/2023 - 10/30/2023	Asset-Liability Management

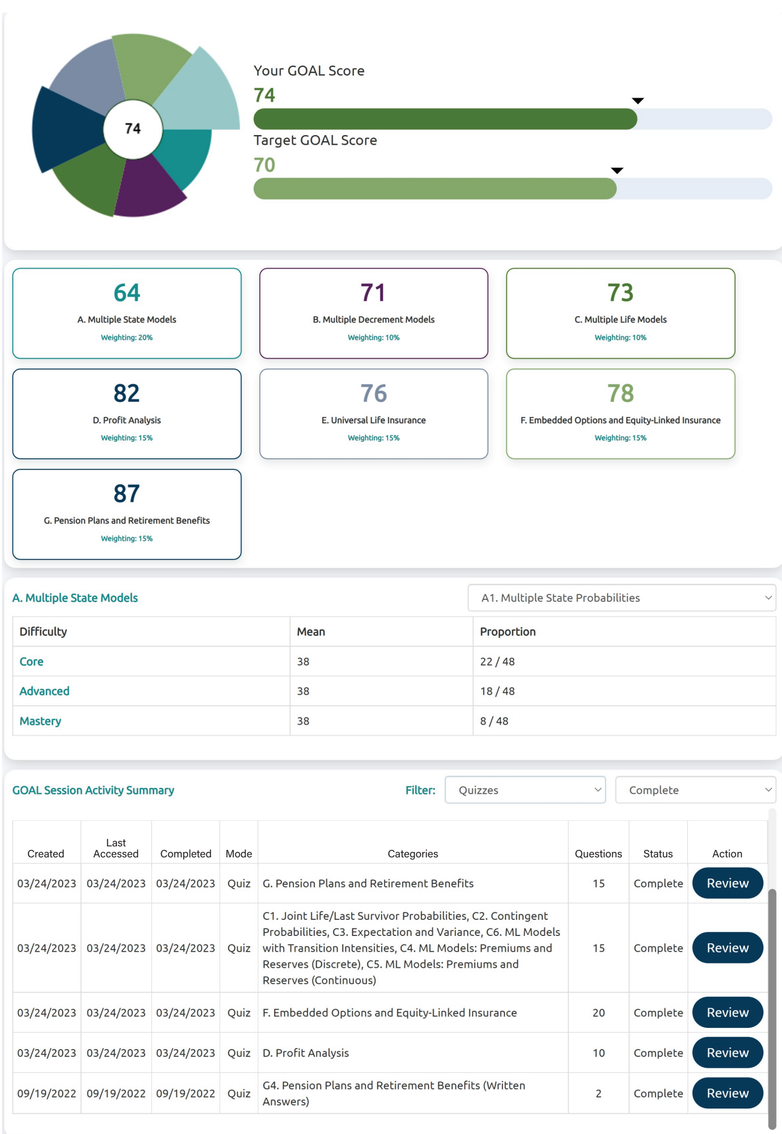
**Available standalone, or included with the Study Manual Program Video Bundle*

Track your exam readiness with GOAL Score!

Available for P, FM, FAM, FAM-L, FAM-S, ALTAM, ASTAM, MAS-I, MAS-II, & CAS 5

GOAL Score tracks your performance through GOAL Practice Sessions, Quizzes, and Exams, resulting in an aggregate weighted score that gauges your exam preparedness.

By measuring both your performance, and the consistency of your performance, **GOAL Score** produces a reliable number that will give you confidence in your preparation before you sit for your exam.



If your GOAL Score is a 70 or higher, you are well-prepared to sit for your exam!

See key areas where you can improve.

Detailed performance tracking.

Quickly return to previous sessions.

Contents

Appendices 921

Preface

Welcome to Exam FAM-L!

Syllabus

The Fall 2022 syllabus is posted at the following URL:

https://www.soa.org/497029/globalassets/assets/files/edu/2022/2022-10-exam-fam-syllabus.pdf

The topics are

1. Survival models
2. Insurances
3. Annuities
4. Premiums
5. Reserves
6. Estimating mortality rates

The textbook for the course is *Actuarial Mathematics for Life Contingent Risks* third edition. This is a college-style textbook. It is oriented towards practical application rather than exam preparation. Almost all exercises require use of spreadsheets or derivation of formulas.

The syllabus splits the material into five broad topics and states percentage ranges for the topics. In the following table, I've doubled the percentages to account for FAM-L being half an exam.

Topic	Weight	Lessons in Manual
Insurance Coverages and Retirement Financial Security Programs	5–15%	2
Mortality Models	15–25%	3–9
Parametric and Non-Parametric Estimation	10–20%	10–13
Present Value Random Variables for Long-Term Insurance Coverages	20–30%	14–25
Premium and Policy Value Calculation for Long-Term Insurance Coverages	25–35%	26–41

For a 20-question exam, each 5% represents one question.

Other downloads from the SOA site

Tables

Download the tables you will be given on the exam. They will often be needed for the exercises. They are currently posted at

https://www.soa.org/4a1b80/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-tables.pdf

The tables include the Standard Ultimate Life Table with insurance functions, and some interest functions. The tables do not include a standard normal distribution table. Instead, you will use a Prometric calculator at the exam, which will provide standard normal distribution and the inverse of that function to 5 decimal places.

Another set of tables, the tables from the former MLC, will be useful if you wish to work on pre-2018 exam questions that use the Illustrative Life Table. You can find it at https://www.soa.org/Files/Edu/edu-2013-mlc-tables.pdf

However, I have converted all pre-2012 exam questions to use the Standard Ultimate Life Table, and the SOA converted questions from 2012 and later when they incorporated them in their sample questions. So it is unlikely you'll need the Illustrative Life Table.

Notation and terminology note

The notation and terminology note is at

https://www.soa.org/4a2744/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-notation.pdf

In almost all cases, the exam uses the terminology of *Actuarial Mathematics for Life Contingent Risks*. This manual uses the terminology that will be used on the exam.

The textbook uses the unusual name "policy value" for "reserve". On LTAM and earlier exams, the exams used the term "reserve", but the notation and terminology note states that "policy value" will be used on FAM-L for the expected value of future loss. "Reserve" will only be used for the capital a company puts aside to cover future losses.

Sample questions

Sample questions are at

https://www.soa.org/4a3519/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-quest.pdf

and their solutions are at

https://www.soa.org/4a3522/globalassets/assets/files/edu/2022/2022-10-exam-fam-l-sol.pdf

At the end of each lesson in this manual, you will find a list of multiple-choice sample questions related to the material of the lesson if there are any. The questions and solutions themselves are not included in this manual. However, solutions to the original exam questions (which, for MLC, used the Illustrative Life Table) are in Appendix B.

Old exam questions in this manual

There are about 380 original exercises in the manual and about 700 old exam questions. The old exam questions come from old Part 4, Part 4A, Course 150, Course 151 exams, 2000-syllabus Exam 3, Exam C, Exam M, and Exam MLC. However, very few questions from the 2012 and later MLC exams are given in the exercises, so you may use those exams or the SOA sample questions as final practice.

SOA Part 4 in 1986 had morning and afternoon sessions. I indicate afternoon session questions with "A". The morning session had the more basic topics (through reserves), while the afternoon session had advanced topics (multiple lives, multiple decrements, etc.) Both sessions were multiple choice questions.

SOA Course 150 from 1987 through 1991 had multiple choice questions in the morning and written answer questions in the afternoon. Since LTAM will include written answer questions, I've included all applicable written answer questions in the exercises.

The CAS Part 4A exams awarded varying numbers of points to questions; some are 1 point and some are 2 points. The 1 point questions are probably too easy for a modern exam, but they'll give you a little practice. The pre-1987 exams probably were still based on Jordan (the old textbook), but the questions I provided, while ancient, still have value. Similarly, the cluster questions on SOA Course 150 in the 1990s generally were awarded 1 point per question.

Although the CAS questions are limited to certain topics, are different stylistically, and are easier, they are a good starting point.

Course 151 is the least relevant to this subject. I've only included a small number of questions from 151 in the first lesson, which is background.

Back in 1999, the CAS and SOA created a sample exam for the then-new 2000 syllabus. This exam had some questions from previous exams but also some new questions, some of them not multiple choice. This sample exam

Table 1: 9 Week Study Schedule for Exam LTAM

Subject	Lessons	Study Period	Hard/Long Lessons	Easy/Short Lessons
Types of Long Term Products	2	0.5 weeks		
Survival Distributions	3–9	2 weeks	4,8	3
Estimation	10–13	1 week		10
Insurances	14–19	1 week	14,18	19
Annuities	20–25	1 week	20,23	25
Premiums	26–34	1.5 weeks	28,33	
Reserves, Part I	35–37	1 week		
Reserves, Part II	38–41	1 week	39, 41	

was never a real exam, and some of its questions were defective. This sample exam is no longer available on the web. I have included appropriate questions from it. *Whenever an exercise is labeled 1999 C3 Sample, it refers to the 1999 sample, not the current list of sample questions.*

Questions from CAS exams given in 2005 and later are not included in this manual. There is a lot of better practice material available, so in order to make this manual a little less bulky, I do not provide solutions to old CAS 3, 3L, and LC exams.

Questions from old exams are marked xxx:yy, where xxx is the time the exam was given, with S for spring and F for fall followed by a 2-digit year, and yy is the question number. Sometimes xxx is preceded with SOA or CAS to indicate the sponsoring organization. From about 1986 to 2000, SOA exams had 3-digit numbers (like 150) and CAS exams were a number and a letter (like 4A). From 2000 to Spring 2003, the exams were jointly sponsored. There was a period in the 1990s when the SOA, while it allowed use of its old exam questions, did not want people to reveal which exam they came from. As a result, I sometimes had study notes for old exams in this period and could not identify the exam they came from. In such a case, I mark the question aaa-bb-cc:yy, where aaa-bb-cc is the study note number and yy is the question number. Generally aaa is the exam number (like 150), and cc is the 2-digit year the study note was published.

Characteristics of this exam

The exam will have 20 multiple choice questions worth 2 points apiece You will be given 1.75 hours to complete the exam, or 5.25 minutes per question.

Ttere is no penalty for guessing. Fill in all questions regardless of whether you have time to work out the question or not—you lose nothing and you may be lucky!

The answer choices on SOA exams are almost always specific answers, not ranges.

Study schedule

Although this manual seems huge, much of it is exercises and practice exams. You do not have to do every exercise; do enough to gain confidence with the material. With intense studying, you should be able to cover all the material in 4 months.

It is up to you to set up a study schedule. Different students will have different speeds and different constraints, so it's hard to create a study schedule useful for everybody. However, I offer a sample 9-week study schedule, Table 1, as a guide. The amount of time you spend on this lesson depends on the strength of your probability background. You may decide to skip it and refer to it as needed.

The study schedule lists lessons that are either long or hard, as well as those that are short or easy or just background, so that you may better allocate your study time within the study periods provided for each subject.

Acknowledgements

I would like to thank the SOA and CAS for allowing me to use their old exam questions. I'd also like to thank Harold Cherry for suggesting this manual and for providing three of the pre-2000 SOA exams and all of the pre-2000 CAS exams I used.

The creators of TEX, LATEX, and its multitude of packages all deserve thanks for making possible the professional typesetting of this mathematical material.

Errata

Please report all errors you find in these notes to the author. You may send them to the publisher at mail@studymanuals.com or directly to me at errata@aceyourexams.net. Please identify the manual and edition the error is in. This is the 1st edition of the Exam FAM-L manual.

An errata list will be posted at errata.aceyourexams.net. Check this errata list frequently.

Flashcards

Many students find flashcards a useful tool for learning key formulas and concepts. ASM flashcards, available from the same distributors that sell this manual, contain the formulas and concepts from this manual in a convenient deck of cards. The cards have cross references, usually by page, to the manual.

Lesson 2

Introduction to Long Term Insurance

Reading: *Actuarial Mathematics for Life Contingent Risks* 3rd edition

A long-term insurance contract is a contract in which the insurer makes commitments that extend for more than one year. Generally the insurer offers insurance for a long period of time at a guaranteed premiums, or at least limits the degree to which the premium can be changed.

Life insurance is the long-term insurance contract we will concentrate on for most of this course. It is the simplest long-term insurance since the only contingency to consider is death. Any long-term contract has to take death into account; at the very least, death terminates the contract. If other contingencies are involved (like disability or illness), the contract will be more complicated than a life insurance contract.

The first chapter of the textbook is an introduction to long-term insurance. It discusses the features of these insurances, but not the mathematics. The following is a summary of the chapter. While I've tried to include anything important, you may want to read the chapter in the textbook to assure yourself that you have learned every little piece of trivia that could be asked on the exam.[1] [2]

Life insurance pays a lump sum, known as a *sum insured*, upon death. (In the U.S., this amount may also be called the face amount of the policy.) The insured pays a premium at the beginning of the period, often the year, for this insurance. Originally, a group of individuals of a specific age or in a specific age group paid an amount so that the total amount collected equalled the total benefits paid. This is called ***assessmentism***. Group life insurance still uses assessmentism. But for individuals, this method means that premiums increase with age. People are likely to withdraw from the group when the premium gets too high.

This led to the level premium insurance contract. In this contract, the same premium is paid each year. These contracts are more complex since the higher premiums collected in the earlier years must be invested to pay for the higher benefits of the later years.

In order to purchase life insurance, one must have ***insurable interest*** in the life insured. One has to be related to the insured life, or have a close business relationship with that person. Otherwise the insurance is a wager on someone's death, which is not desirable.

Traditional insurance contracts Traditional insurance contracts have fixed premiums and fixed benefits. Life insurance provides a benefit upon death; annuities provide a periodic payment while the annuitant is alive. For life insurance products, premiums are payable periodically (annually, semi-annually, quarterly, or monthly) throughout the life of the contract. However, the premium payment period often is limited to a certain number of years or until the insured reaches a certain age, because older insureds may be less able to pay the premium. For annuities, a single premium may be paid. Otherwise, periodic premiums are payable during a deferral period; the annuity benefits begin after the deferral period. You would not typically have money going both ways at the same time—premiums payable while the annuitant is receiving benefits.

Examples of traditional insurance contracts are

Term insurance **Term insurance** pays a benefit if the policyholder dies within a fixed amount of time. It is useful for insuring a family at a low cost, or insuring a business against losses due to death of an employee. In the latter case, the company pays the premiums and receives the benefits. This type of insurance may be called *key person insurance* or *COLI*–company owned life insurance.

[1] Examples of trivia I've not included are:

1. In the UK, when is it called insurance and when is it called assurance?
2. Which insurance rider inspired a Hollywood feature?

[2] in this lesson, footnotes are my own comments on the textbook, and occasionally modify the textbook. They are purely to give you the situation in the U.S.; the textbook tends to reflect the international situation. On exams do not use anything from these footnotes!

Other types of term insurance are

1. Decreasing term insurance to pay off the balance of a mortgage.
2. **Renewable term insurance** allows renewal at the expiry date, at an increased premium.
3. **Convertible term insurance** allows conversion of the policy to whole life.

Whole life **Whole life insurance** pays a benefit whenever the policyholder dies. Premiums are level. The policy has a cash value that is paid if the policyholder surrenders the policy.

In the U.S., non-forfeiture laws require cash surrender values to be offered on whole life insurance. These cash values increase over the years of the policy, but are less than the face amount.[3]

Some uses of whole life insurance are:

1. Payment of funeral expenses.
2. Payment of estate taxes.
3. Saving vehicle for younger lives.

Endowment insurance **Endowment insurance** pays a benefit if the policyholder dies within a certain period. If the policyholder survives to the end of the period, the benefit is paid at that point. It has cash values. Endowment insurance is not offered in North America or the U.K. because the investment component does not compete with other investments; the returns are low and the investment is not flexible.[4] But endowment insurance is popular in developing nations.

Joint life/multiple life insurance This type of insurance pays upon the first death or the last death in a group of individuals. **Joint life** is a policy on husband and wife; companies may insure their officers using multiple life insurance. According to the textbook first-to-die policies are more common than last-to-die policies.[5]

Participating insurance Whole life insurance must make interest assumptions into the distant future. In order not to lose money, these assumptions are conservatively low. But the product is then unattractive. To compensate, **participating insurance** (par insurance for short) pays non-guaranteed dividends based on the excess of the rate earned on investments over the assumption. These dividends are known as bonuses outside North America.

In North America, policyholders may choose to receive these dividends in many forms, such as

Cash refunds Dividend is paid as cash.

Premium reductions Dividend is offset against premium. This is really no different from a cash refund.

Increased paid up insurance The death benefit is increased.[6]

In the U.K., dividends are used only to increase the death benefit, the third option above. The dividends are called ***reversionary bonuses***, and come in three variations[7]:

simple reversionary bonuses Applied to original face amount only

compound reversionary bonuses Applied to sum of original face amount and prior bonuses

super-compound reversionary bonuses Two different bonus rates, one for original face amount, another for prior bonuses.

---

[3] If they were greater than the face amount, a policyholder on his death bed would surrender the policy.

[4] And importantly, the tax laws offer very unfavorable treatment to these policies (but the book doesn't mention this).

[5] But in the U.S., last-to-die products on husband/wife are especially popular as a method of paying estate taxes, due to their low premiums. Generally estates pass tax-free to a spouse, so the benefit is not needed until the second death.

[6] Other options not mentioned in the textbook are (1) Leave the dividend with the company to accumulate with interest, (2) purchase one-year term insurance; in other words, increase the death benefit for one year only

[7] In the U.S., dividends used to purchase paid up additions follow the "super-compound reversionary bonus" style.

Terminal bonuses[8] are paid on policies that terminate due to death or surrender.
Advantages/disadvantages of cash dividends versus reversionary bonuses are

1. Cash dividends allow policyholders to use them to pay premiums, which may be helpful if policyholder is short of cash.
2. Cash dividends may be taxable.
3. Cash dividends are simpler to understand.
4. Reversionary bonuses are more consistent with the purpose of the policy.
5. Companies may only give some portion of reversionary bonuses to policyholders who surrender, which is unfair to them.[9]
6. Reversionary bonuses make it easier for companies to smooth their dividends.
7. Cash dividends are expensive to operate.
8. Cash dividends require liquidity for the insurer, restricting its investments somewhat.

Additional benefits that policies may provide

Policy loans For policies with cash values, a **policyholder may take a loan against the cash value**. This loan would be deducted against the benefit if it is not repaid before death.

Accelerated death benefits Under this rider, the **death benefit is paid upon terminal illness**.

Accidental death benefit Under this rider, also known as *double indemnity*, **twice the death benefit is paid if death is due to accident**.

Disability waiver Under this rider, the **premium is waived upon disability**.

Family income This rider is term insurance. Under this rider, upon death, **a fixed payment is made each month until the expiry of the rider**. For example, if the rider is for 10 years and the insured dies within 8 years, the policy pays a benefit for 2 years. Thus this is decreasing term insurance. According to the textbook, the base policy is a term insurance for the same number of years.

Reasons the design of life insurance contracts has changed recently

1. Competition with mutual funds and banks for policyholders' savings. Thus insurance products have investment components.
2. Changing demographics and lifecycles.
3. Developments in financial science and technology. Modern financial science improves risk management techniques. Computer technology allow more complex products.
4. Better informed customers. Smarter customers require more competitive products.

Nontraditional insurance contracts Here are examples of non-traditional products. We do not discuss non-traditional products any further in this course

1. **Universal Life**. This product has flexible premiums. The benefit payable upon death may also be adjustable.

   Premiums go into a notional account. It is notional only; there are no designated assets. The insurer subtracts mortality and expense charges and credits interest. The credited interest rate may change from time to time, but cannot be less than 0%. Generally charges and credits are made monthly. The policy stays in force as long as the account balance is positive. In the earlier years, there is a surrender charge if a policyholder surrenders the policy.

[8]In North America they're called terminal dividends.
[9]In the U.S., this is not allowed. Once a policyholder gets a paid-up addition, it cannot be taken back.

2. **Unitized-with-profit.** This is a UK product. In the UK, products that allow policyholders to share in the company's profits (which are called "participating" in the US) are called "with-profit". In unitized-with-profit contracts, the premium purchases units of investments, which earn reversionary bonuses, but cannot decrease in value.

3. Equity-linked insurance. Two types[10]:

   (a) **Variable annuity** (called "unit-linked" in the UK) invests premiums in mutual funds. Credited amounts may be positive or negative, depending on investment performance.

   (b) **Equity-indexed annuity** has guaranteed returns, and the policyholder receives a portion of the growth on an index (e.g. a stock index) if greater than the guarantee.

Distribution methods Most insurance is sold through brokers. Brokers are paid commissions, which are a percentage of premium. Usually the first year commission is a higher percentage of premium than renewal commissions. This is referred to as a *front end load*.

Direct marketing is an alternative to brokers. Typically policies sold by direct marketing are for small face amounts and are not as heavily underwritten as broker-sold policies. Hence mortality is higher, but expenses are lower both due to simplified underwriting and lack of commissions. Often such policies are for funeral expenses or credit insurance—insurance paying off a loan if the insured dies.

Underwriting classes When a company sells insurance, it usually **underwrites** the policyholder; it determines how healthy the policyholder is and sets premiums based on the policyholder's underwriting class. Typically the underwriting classes are:

1. Preferred: low risk factors, no smoking or alcohol abuse, no high-risk occupations or hobbies. Preferred classes are common in North America, but not in the rest of the world.

2. Normal[11]: some risk factors, but insurable at standard rates.

3. Rated: Elevated risk factors, higher than standard rates must be charged.

4. Uninsurable: Too risky, not insurable at any price.

For life insurance, there is adverse selection by policyholders; the sicker one is, the more likely one is to buy life insurance, and for higher amounts. Underwriting therefore is stricter the higher the sum insured is. Underwriting may only require a policyholder statement for a small amount, but will require a physician's statement for a larger amount and may require examination by the company's doctor for very large amounts.

Types of annuities An annuity makes a periodic payment to the purchaser of the annuity. Types of annuities are:

1. Single Premium Deferred Annuity (SPDA). Policyholder pays a single premium. Annuity commences payments on some future date.

2. Single Premium Immediate Annuity (SPIA). Policyholder pays a single premium and annuity payments commence immediately. For example, if payments are monthly, the first payment is one month after the premium is paid.

3. Regular Premium Deferred Annuity (RPDA). Policyholder pays premium on a regular basis until annuity payments commence.

These three types categorize annuities based on when the premiums and benefits are paid. The next three types categorize when benefits are paid for annuities on two lives.

4. Joint life annuity. Annuity makes payments only when both are alive.

[10]For unknown reasons, the textbook discusses annuities here rather than insurances.
[11]Called "standard" in the U.S.

5. Last survivor annuity. Annuity makes payments when at least one is alive.

6. Reversionary annuity. Annuity makes payments to second life after first life dies.

A *guaranteed annuity* is one that makes payments for a period regardless of whether the annuitant is alive. Another name for this annuity is a "certain and life annuity".

Life annuities during their payment period do not have a surrender value. Allowing surrender of such annuities would lead to adverse selection, since individuals who believe they will die soon will surrender their annuities. The pricing of annuities assumes that some people will die earlier, leaving funds to pay annuitants who live longer.

Annuities are not underwritten. Insurance companies make more money if the annuitant dies earlier. It is generally not possible to collect evidence that an annuity purchaser will not live too long.[12]

Disability income insurance **Disability insurance** pays a monthly benefit while the policyholder is disabled. Benefit is a proportion of salary, generally 50–70%, to encourage return to work. Insurance may continue until retirement.

Some time provisions of disability insurance are:

Waiting period , or *elimination period*. Benefits only begin after insured is disabled for this period. Typically 30, 60, 180, or 365 days.

Benefit period . Benefits are payable for at most a certain amount of time. Time is measured from when benefits start, after the waiting period. Typical options are 2 years, 5 years, or to age 65.

Off period . This is the amount of time that must elapse between two disability periods before they are considered separate. This may work both for and against the policyholder; if the off period has not passed, there is no new waiting period, but there is no new benefit period either; the second period's benefits would only be for the remainder of the benefit period minus the first disability's period.

Some other characteristics of disability insurance are:

- If the policyholder can do some work but not full-time, the policyholder may be eligible for a lower benefit for partial disability.
- Benefit may be reduced if policyholder receives benefits from other sources, like the government.
- Own job versus any job. Own job insurance pays benefits if the disabled person can't return to his job, even if other work is available. Any job insurance only pays if the disabled person is so ill that he cannot perform any job. Own job insurance is, or course, more costly.[13]
- Employers may purchase disability insurance for employees. Rates are lower than individual policies due to
  1. No adverse selection
  2. Economies of scale
  3. No risk of non-payment of premium
- Some policies have *return to work assistance*. This may pay for retraining or other items that help the disabled person return to work.

Long term care insurance This insurance pays if someone is ill enough to require a home health aid or a nursing home. Generally a 90-day waiting period. Benefits are triggered when one cannot perform 2 or 3 (depending on the policy) **Activities of Daily Living** (ADLs).

There are 6 ADLs:

1. Bathing

[12] But some companies have experimented with offering lower prices for annuities to smokers.
[13] Some disability insurance may be "own job" for a period like 5 years, and "any job" after that.

2. Dressing
3. Eating
4. Toileting (ability to go to/from toilet)
5. Continence (bladder/bowel control)
6. Transferring (ability to go from chair to bed to chair; distinguished from toileting)

Severe cognitive impairment may also trigger benefits.

Some other features are:

- Just like disability, there is a benefit period which may be 2–5 years, or, unlike disability, may be for life.
- Just like disability, there is an off period.
- Benefit payments may be on a reimbursement approach subject to a maximum, or may be a fixed amount per month.

Hybrid life insurance/LTC policies may use one of two approaches:

Return of premium approach The excess of premiums over benefit payments is added to the death benefit.[14]

Accelerated benefit approach The total LTC benefit cannot exceed the sum insured of the life insurance. The LTC benefits paid are subtracted from the death benefit.

The discussion so far relates to U.S. and Canada. In other countries:

France LTC is popular and cheaper because

1. Trigger for most policies is "severe dependency", meaning bed- or chair-bound, stricter than "2 ADL" condition.
2. Policies mostly bought as group insurance.
3. Policies bought at younger ages.
4. Lower average benefits.

Payment is fixed annuity.

Germany LTC provided by government social insurance. Additional benefits may be obtained through private LTC insurance. May opt out of government plan (and not pay taxes for it) and use private insurance.

Payment is fixed annuity.

Japan Available stand-alone or with whole life insurance. Benefits may increase with increased dependency.

UK LTC not offered. Instead, there is an *immediate needs annuity* that one purchases with a single premium when moving to a nursing home. Benefits are a fixed annuity paid directly to nursing home. Insurance company may assume higher-than-standard mortality.

Critical illness/chronic illness insurance **Critical illness insurance** pays a lump sum benefit upon a severe condition, such as cancer or heart disease, and then expires.

Chronic illness insurance pays a benefit upon an illness from which one may not recover. Benefit may be lump sum or annuity.

Either of these two may be an **accelerated death benefit rider** to a life insurance contract.

[14] I think that for most such policies, there is no other death benefit; the death benefit is just the return of premiums minus benefits.

Forms of ownership of insurance companies

1. **Mutual**. Has no stockholders. Profits are distributed to for-profit (participating) policyholders.
2. **Proprietary**[15]. Profits are distributed to shareholders. May have for-profit (participating) policies, and then those policyholders would also get share of profit.

Continuing care retirement communities **Continuing care retirement communities** (CCRCs) are communities for seniors who need varying amounts of assistance. They have three levels of support:

1. Independent living units (ILUs) for residents who do not need extensive help. They may offer housekeeping, transportation to shopping, and similar services.
2. Assisted living units (ALUs), that provide more extensive non-medical help, such as cooking and laundering.
3. A skilled nursing facility (SNF) for those needing medical care. This looks more like a hospital.

Some facilities may also have memory care units (MCU) for residents with dementia.

Funding is done through

1. *Full life care*, with a large entry fee and guaranteed monthly payments only increasing with cost of living adjustments. All costs are covered.
2. *Modified life care*, with a lower entry fee and with monthly payments. Monthly fees increase if the resident moves to the ALU, MCU, or SNF, but these increases are less than the full cost.
3. *Fee for service*. Here payments are made for all services provided at the market rates. Entry fees and monthly payments (at least initially) are lowest.

For full and modified life care contracts:

1. They are insurance contracts in the sense that the CCRC assumes the risk of the costs of higher level care. Residents entering with these contracts must be underwritten and healthy enough to qualify. Residents must be able to initially live in the ILU.
2. Some contracts provide a partial refund upon exit.
3. Some CCRCs offer (partial) ownership of the ILU. The resident purchases the unit and upon death or exit it is sold and the resident gets some of the proceeds.

Average age of entry to CCRC in the U.S. is 80. On the average, full life care residents enter at younger ages than modified life care residents, who enter at younger ages than fee-for-service residents.

Structured settlements When a person is injured, the responsible party must compensate for the loss. Compensation includes paying for medical treatment and lost wages. Rather than paying a lump sum, the insurance company or the court often provides a **structured settlement**. This settlement may include a lump sum for medical payments and other immediate expenses and a life annuity for lost wages, ongoing medical expenses, and other expenses. Providing a structured settlement is better than paying a lump sum because the payment pattern of the structured settlement better matches the pattern of the losses— the lost wages and the ongoing medical expenses resemble an annuity. Thus it relieves the injured party of two risks:

1. The investment risk—the risk of not getting an adequate return on a lump sum
2. The dissipation risk—the risk that the lump sum will be spent too fast

[15] Called "stock" in the U.S.

A serious injury may require a whole life annuity, whereas a more minor injury would require a temporary life annuity extending until expected recovery.

Structured settlements are commonly used in Workers Compensation insurance (called workers comp for short), an insurance purchased by an employer to pay benefits to workers injured on the job. They are also commonly used in medical malpractice and motor vehicle accidents.

Usually a structured settlement once determined is final. But after severe injury, there may be a period in which payments are made until the time of maximum mortality improvement, at which point a final settlement is made.

Pensions A pension is a lump sum or a life annuity paid to a retiree by the employer.

The amount of a pension usually varies with the employee's salary and number of years of service.

Pension plans may be defined contribution or defined benefit. In a **defined contribution plan**, the amount set aside each year of employment is specified.

In a **defined benefit plan**, the annual amount paid to the retiree is specified. Typically the annual amount is $n\alpha S$, where n is the number of years of service, α is an accrual rate specified in the plan, and S is the salary base. α is typically 1–2%. The salary base may be

Final salary **The average of salaries in the k years before retirement**.

Career average salary **The average of all salaries earned while in service.**

Career average revalued salaries The average of all salaries with the salaries indexed for inflation.

Defined benefit plans may have *withdrawal benefits*. These are benefits to terminated employees. Typically these would be computed using the $n\alpha S$ formula, and deferred to retirement age.

Defined contribution plans pay a lump sum of the accumulated funds at retirement. They may be used to purchase an annuity.

Old SOA Exam LTAM questions: F18:B4(a), F19:B4(a), S21-1:1,B3(a), F21-A:1, F21-B:1

Multiple choice sample questions: 1.1, 1.2

Part VII

Practice Exams

Here are 12 practice exams to help you test your knowledge, and to pinpoint areas you are weak in so you will know what to review.

The number of questions on each of the five major topics of Exam FAM-L are in line with the syllabus. The questions are randomly arranged, both in terms of topic and in terms of difficulty. On real exams (or at least the ones they've released), there are some easy questions at the beginning and an easy question at the end, but these exams don't follow that rule.

Practice Exam 1

1. A life age 60 is subject to Gompertz's law with $B = 0.001$ and $c = 1.05$.

Calculate $e_{60:\overline{2}|}$ for this life.

(A) 1.923 (B) 1.928 (C) 1.933 (D) 1.938 (E) 1.943

2. Your company sells whole life insurance policies. At a meeting with the Enterprise Risk Management Committee, it was agreed that you would limit the face amount of the policies sold so that the probability that the present value of the benefit at issue is greater than 1,000,000 is never more than 0.05.

You are given:

(i) The insurance policies pay a benefit equal to the face amount b at the moment of death.
(ii) The force of mortality is $\mu_x = 0.001(1.05^x)$, $x > 0$
(iii) $\delta = 0.06$

Determine the largest face amount b for a policy sold to a purchaser who is age 45.

(A) 1,350,000 (B) 1,400,000 (C) 1,450,000 (D) 1,500,000 (E) 1,550,000

3. For an annual premium 2-year term insurance on (60) with benefit b payable at the end of the year of death, you are given

(i)

t	p_{60+t-1}
1	0.98
2	0.96

(ii) The annual net premium is 25.41.
(iii) $i = 0.05$.

Determine the revised annual net premium if an interest rate of $i = 0.04$ is used.

(A) 25.59 (B) 25.65 (C) 25.70 (D) 25.75 (E) 25.81

4. For a fully discrete 25-year term life insurance on (45) with face amount 200,000, you are given:

(i) Mortality follows the Standard Ultimate Life Table.
(ii) Deaths are uniformly distributed between integer years.
(iii) Gross premium payable quarterly is 130.
(iv) Per premium and per policy expenses are

	Percent of Premium	Per Policy
First year	60%	250
Renewal	5%	30

(v) Per premium expenses are payable when premiums are payable.
(vi) Per policy expenses are payable at the beginning of each year.
(vii) Cost of settling a death claim is 100.
(viii) $i = 0.05$.

Calculate the gross premium reserve at time 15.

(A) 4817 (B) 4822 (C) 4826 (D) 4892 (E) 4896

5. Endowment insurance is no longer offered by major insurers in North America or the UK.
Consider the following reasons:

I. The product has low returns.
II. The product is not flexible.
III. There are onerous tax provisions on the product.

State which of these reasons is given in *Actuarial Mathematics for Life Contingent Risks*.

(A) None (B) I and II only (C) I and III only (D) II and III only
(E) The correct answer is not given by (A) , (B) , (C) , or (D) .

6. A study is performed on number of days required to underwrite a policy. The results of the study are:

Number of Days	Number of Policies
(0,10]	11
(10,20]	x
(20,50]	y

An ogive is used to interpolate between interval boundaries.
You are given:

(i) $\hat{F}(15) = 0.35$
(ii) $\hat{f}(15) = 1/30$

Determine x.

(A) 16 (B) 18 (C) 20 (D) 22 (E) 24

Exam questions continue on the next page . . .

7. You are given:

(i) Z_1 is the present value random variable for a 10-year term insurance paying 1 at the moment of death of (45).
(ii) Z_2 is the present value random variable for a 20-year deferred whole life insurance paying 1 at the moment of death of (45).
(iii) $\mu = 0.02$
(iv) $\delta = 0.04$

Calculate $\text{Cov}(Z_1, Z_2)$.

(A) −0.042 (B) −0.028 (C) −0.023 (D) −0.015 (E) −0.009

8. You are given:

(i) For a cohort of 100 newly born children, the force of mortality is constant and equal to 0.01.
(ii) Birthday cards are sent each year to all lives in the cohort beginning on their 80^{th} birthdays, for as long as they live.

Determine the expected number of birthday cards each member of this cohort receives.

(A) 44.7 (B) 44.9 (C) 45.2 (D) 45.5 (E) 45.7

9. A special 9-year term insurance on (x) pays the following benefit at the end of the year of death:

Year of death t	1	2	3	4	5	6	7	8	9
Benefit b_t	1	2	3	4	5	4	3	2	1

$(DA)^{1}_{x:\overline{n}|}$ denotes the expected present value of a decreasing term insurance that pays a benefit of $n+1-k$ at the end of the year if death occurs in year k, $1 \le k \le n$.

You are given the following expected present values for increasing and decreasing term insurances:

n	$(IA)^{1}_{x:\overline{n}\vert}$	$(DA)^{1}_{x:\overline{n}\vert}$
4	0.593	0.628
5	0.848	0.923
9	1.970	2.513
10	2.219	2.986

Determine the expected present value of the special term insurance.

(A) 0.7 (B) 0.8 (C) 1.3 (D) 1.4 (E) 1.8

10. For a fully continuous whole life insurance of 1000 on (x):

(i) The gross premium is paid at an annual rate of 25.
(ii) The variance of future loss is 500,000.
(iii) $\delta = 0.06$

Employees are able to obtain this insurance for a 20% discount.

Determine the variance of future loss for insurance sold to employees.

(A) 320,383 (B) 301,261 (C) 442,907 (D) 444,444 (E) 456,253

Exam questions continue on the next page . . .

11. You are given that $A_x = 0.4 + 0.01x$ for $x < 60$.

A fully discrete whole life insurance on (30) pays a benefit of 1 at the end of the year of death.

Calculate the net premium reserve at time 20 for this insurance.

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) $\frac{3}{4}$

12. You are given the following statements regarding disability insurance.

I. "Own job" insurance tends to be cheaper than "any job" insurance.

II. For a policy with benefit period to 65, longer off periods make the insurance more expensive.

III. The cost of a policy increases as the benefit period increases.

(A) None of I, II, or III is true
(B) I and II only
(C) I and III only
(D) II and III only
(E) The answer is not given by (A), (B), (C), or (D)

13. In a mortality study, the cumulative hazard function is estimated using the Nelson-Åalen estimator. There are initially 41 lives. There are no censored observations before the first time of deaths, $t_{(1)}$.

The number of deaths at time $t_{(1)}$ is less than 6.

$\widehat{\mathrm{Var}}\big(\hat{H}(t_{(1)})\big) = 0.000580$.

Determine the number of deaths at time $t_{(1)}$.

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. For a fully discrete 20-year deferred whole life insurance of 1000 on (50), you are given:

(i) Premiums are payable for 20 years.
(ii) The net premium is 12.
(iii) Deaths are uniformly distributed between integral ages.
(iv) $i = 0.1$
(v) ${}_9V = 240$ and ${}_{9.5}V = 266.70$.

Calculate ${}_{10}V$, the net premium reserve at the end of year 10.

(A) 272.75 (B) 280.00 (C) 281.40 (D) 282.28 (E) 282.86

Exam questions continue on the next page . . .

15. A mortality study begins with 2 lives. You are given the following excerpt of the study data:

j	$t_{(j)}$	Deaths at $t_{(j)}$	Exits in $(t^+_{(j)}, t^-_{(j+1)})$ (censored)	Entrants in $(t^+_{(j)}, t^-_{(j+1)})$ (truncated)
0	0		0	1
1	3.1	1	1	2
2	4.0	1	1	5
3	5.2	1	2	0
4	6.2	1	0	0
5	8.4	1	0	0

Calculate the Nelson-Åalen estimate of $S(7)$.

(A) 0.23 (B) 0.25 (C) 0.27 (D) 0.29 (E) 0.31

16. A life age 90 is subject to mortality following Makeham's law with $A = 0.0005$, $B = 0.0008$, and $c = 1.07$. Curtate life expectancy for this life is 6.647 years.
Using Woolhouse's formula with three terms, compute complete life expectancy for this life.

(A) 7.118 (B) 7.133 (C) 7.147 (D) 7.161 (E) 7.176

17. You are given that $\mu_x = 0.002x + 0.005$.
Calculate $_{5|}q_{20}$.

(A) 0.015 (B) 0.026 (C) 0.034 (D) 0.042 (E) 0.050

18. For a temporary life annuity-due of 1 per year on (30), you are given:

(i) The annuity makes 20 certain payments.
(ii) The annuity will not make more than 40 payments.
(iii) Mortality follows the Standard Ultimate Life Table.
(iv) $i = 0.05$

Determine the expected present value of the annuity.

(A) 17.79 (B) 17.83 (C) 17.87 (D) 17.91 (E) 17.95

19. For a mortality table, you are given

(i) Uniform distribution of deaths is assumed between integral ages.
(ii) $\mu_{30.25} = 1$
(iii) $\mu_{30.5} = \frac{4}{3}$

Determine $\mu_{30.75}$.

(A) $\frac{5}{3}$ (B) 2 (C) $\frac{7}{3}$ (D) $\frac{5}{2}$ (E) 3

Exam questions continue on the next page . . .

20. In a mortality study on 5 lives, you are given the following information:

Entry age	Exit age	Cause of exit
62.3	65.1	End of study
63.5	66.0	Withdrawal
64.0	65.7	Withdrawal
64.2	65.5	Death
64.7	67.7	End of study

Assume constant force of mortality between integer ages.

Calculate the maximum likelihood estimate of q_{65}.

(A) 0.261 (B) 0.262 (C) 0.263 (D) 0.264 (E) 0.265

Solutions to the above questions begin on page 923.

Practice Exam 1

1. [**Section 6.2**] By formula (5.2),

$$p_{60} = \exp\left(-0.001(1.05^{60})\left(\frac{0.05}{\ln 1.05}\right)\right) = 0.981040$$
$$_2p_{60} = \exp\left(-0.001(1.05^{60})\left(\frac{1.05^2-1}{\ln 1.05}\right)\right) = 0.961518$$

Then $e_{60:\overline{2}|} = 0.981040 + 0.961518 = \boxed{1.9426}$. **(E)**

2. [**Lesson 17**] The present value of the benefit decreases with increasing survival time, so the 95th percentile of the present value of the insurance corresponds to the 5th percentile of survival time. The survival probability is

$$_tp_{45} = \exp\left(-\int_0^t 0.001(1.05^{45+u})\,\mathrm{d}u\right)$$
$$-\ln {}_tp_{45} = \left.\frac{0.001(1.05^{45+u})}{\ln 1.05}\right|_0^t$$
$$= \frac{0.001(1.05^{45+t}-1.05^{45})}{\ln 1.05}$$

Setting $_tp_{45} = 0.95$,

$$\frac{0.001(1.05^{45+t}-1.05^{45})}{\ln 1.05} = -\ln 0.95$$
$$1.05^{45+t} = (-1000\ln 0.95)(\ln 1.05) + 1.05^{45} = 11.48762$$
$$1.05^t = \frac{11.48762}{1.05^{45}} = 1.27853$$
$$t = \frac{\ln 1.27853}{\ln 1.05} = 5.0361$$

The value of Z if death occurs at $t = 5.0361$ is $be^{-5.0361(0.06)}$, so the largest face amount is $1{,}000{,}000e^{5.0361(0.06)} = \boxed{1{,}352{,}786}$. **(A)**

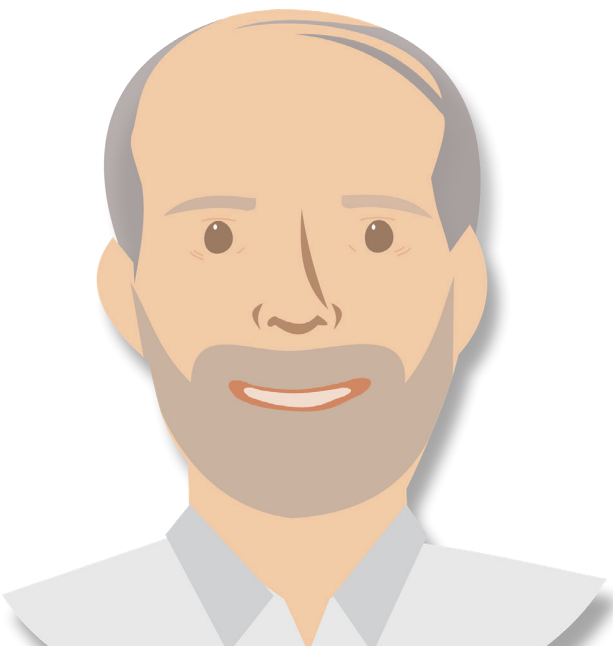
3. [**Lesson 26**] The revised premium for the entire policy is 25.41 times the ratio of the revised premium per unit at 4% to the original premium per unit at 5%.

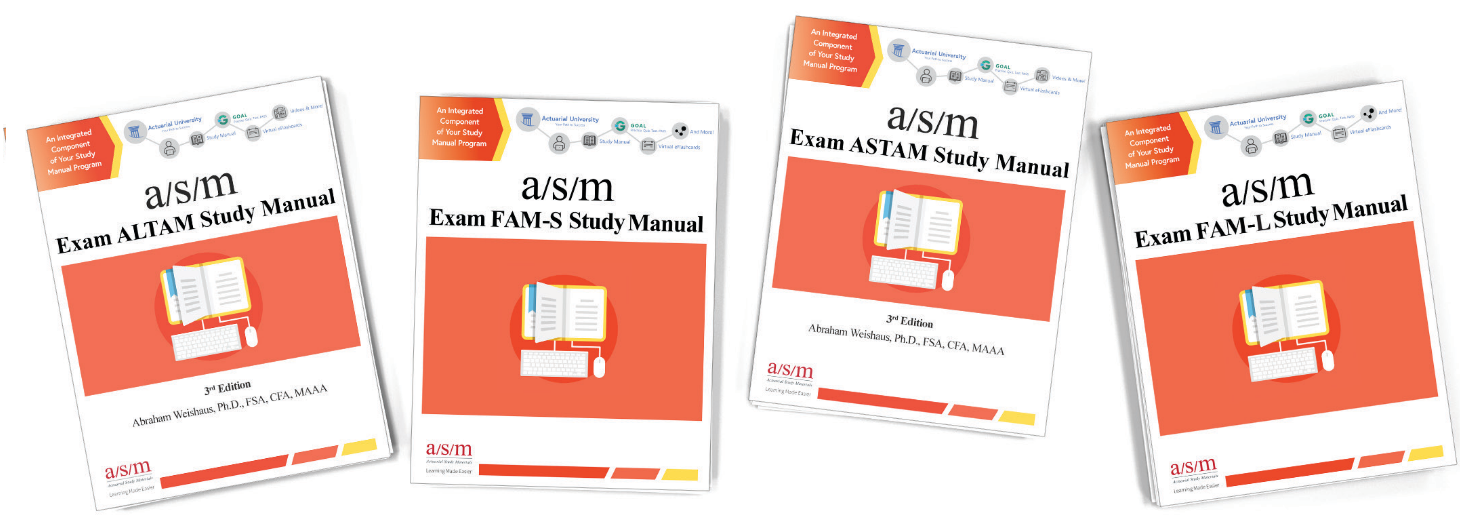
We calculate the original net premium per unit, $P^{1}_{60:\overline{2}|}$.

$$\ddot{a}_{60:\overline{2}|} = 1 + \frac{0.98}{1.05} = 1.93333$$
$$A^{1}_{60:\overline{2}|} = \frac{0.02}{1.05} + \frac{(0.98)(0.04)}{1.05^2} = 0.054603$$
$$P^{1}_{60:\overline{2}|} = \frac{A^{1}_{60:\overline{2}|}}{\ddot{a}_{60:\overline{2}|}} = \frac{0.054603}{1.93333} = 0.028243$$

Now we recalculate at 4%. Call the revised premium $P'^{1}_{60:\overline{2}|}$.

$$\ddot{a}_{60:\overline{2}|} = 1 + \frac{0.98}{1.04} = 1.94231$$
$$A^{1}_{60:\overline{2}|} = \frac{0.02}{1.04} + \frac{(0.98)(0.04)}{1.04^2} = 0.055473$$
$$P'^{1}_{60:\overline{2}|} = \frac{0.055473}{1.94231} = 0.028561$$





a/s/m
Exam ALTAM Study Manual
3rd Edition
Abraham Weishaus, Ph.D., FSA, CFA, MAAA
a/s/m
Exam FAM-S Study Manual
a/s/m
Exam ASTAM Study Manual
3rd Edition
Abraham Weishaus, Ph.D., FSA, CFA, MAAA
a/s/m
Exam FAM-L Study Manual

An Integrated Component of Your Study Manual Program

Actuarial University
Your Path to Success

Study Manual

GOAL
Practice. Quiz. Test. PASS

Virtual eFlashcards

Videos & More!

a/s/m

Exam FAM-S Study Manual

1st Edition, 2nd Printing

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

a/s/m
Actuarial Study Materials
Learning Made Easier

a/s/m

Exam FAM-S Study Manual

1st Edition, 2nd Printing

Abraham Weishaus, Ph.D., FSA, CFA, MAAA

TO OUR READERS:

Please check A.S.M.'s web site at **www.studymanuals.com** for errata and updates. If you have any comments or reports of errata, please e-mail us at **mail@studymanuals.com**.

Welcome to Actuarial University

Actuarial University is a reimagined platform built around a more simplified way to study. It combines all the products you use to study into one interactive learning center.

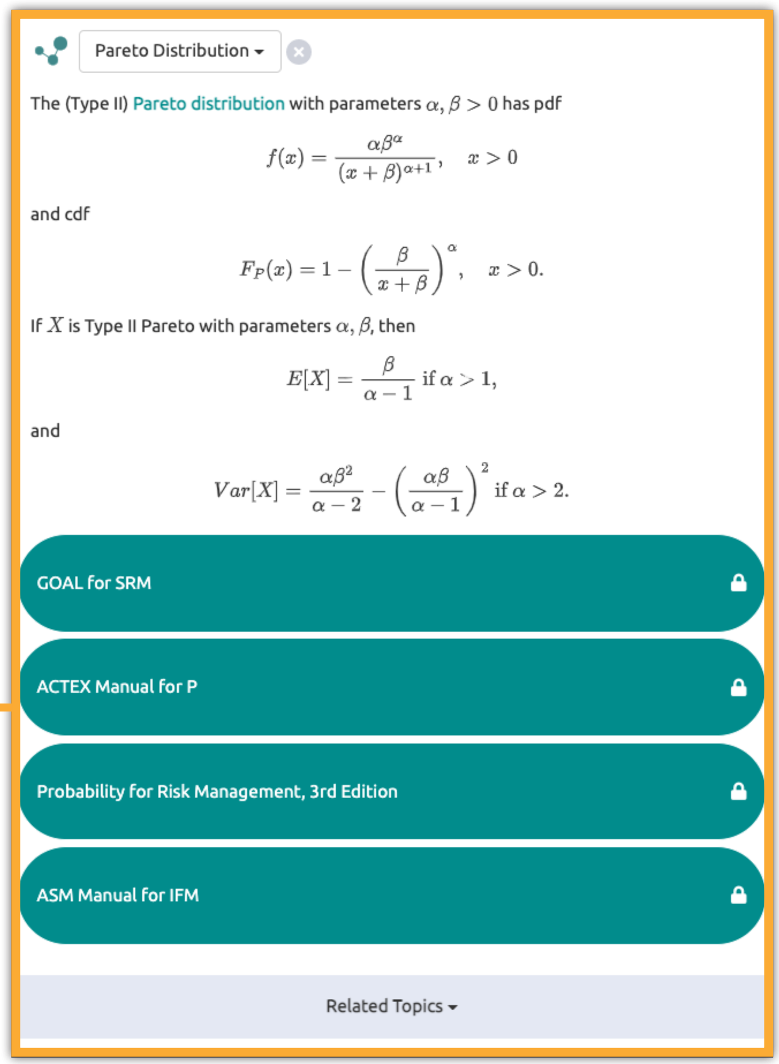
You can find integrated topics using this network icon.
When this icon appears, it will be next to an important topic in the manual! **Click the link** in your digital manual, or **search the underlined topic** in your print manual.

1. Login to: **www.actuarialuniversity.com**

2. Locate the **Topic Search** on your exam dashboard, and enter the word or phrase into the search field, selecting the best match.

3. A topic "**Hub**" will display a list of integrated products that offer more ways to study the material.

4. Here is an example of the topic **Pareto Distribution**:



Within the **Hub**, there will be unlocked and locked products.

Unlocked Products are the products that you own.

GOAL for P →

Locked Products are products that you do not own, and are available for purchase.

Probability for Risk Management, 3rd Edition

Many of Actuarial University's features are already unlocked with your study program, including:

GOAL Practice Tool	**Instructional Videos***
Virtual Flashcards	**Formula & Review Sheet**

GOAL: Guided Online Actuarial Learning

Your Adaptable, Customizable, Online Actuarial Exam Prep Tool.

GOAL is an **eLearning test preparation tool** for students to practice skills learned in class or from independent study. The online platform offers a massive database of **SOA & CAS exam-style problems with detailed solutions**.

GOAL offers **instructor support** as well as **performance reporting and tracking** to monitor your progress. Don't forget to check out **GOAL Score** to see how ready you are to sit for your exam!

Practice. Quiz. Test. Pass!

- 10,000+ Exam-style problems with detailed solutions!
- Adaptive Quizzes
- 3 Learning Modes
- 3 Difficulty Modes

How GOAL Works

**Available standalone, or included with the Study Manual Program Video Bundle*

Practice. Predict. Pass.

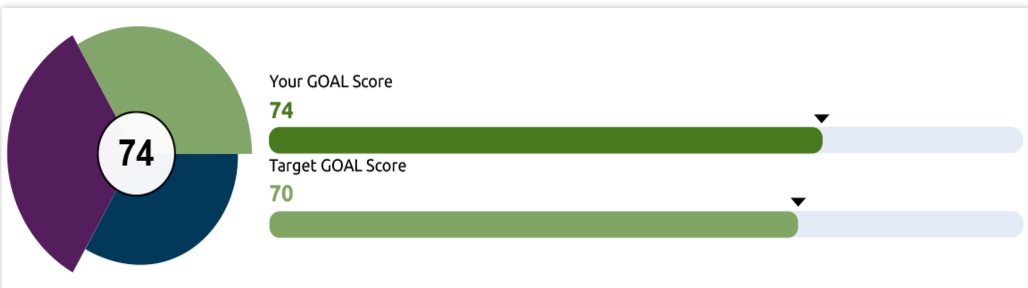
GOAL

Track your exam readiness with GOAL Score!

Available now for P, FM, FAM, ALTAM, ASTAM, SRM, MAS-I, MAS-II, CAS 6C/U

GOAL Score tracks your performance through GOAL Practice Sessions, Quizzes, and Exams, resulting in an aggregate, weighted score that gauges your exam preparedness.

By measuring both **your performance**, and **the consistency of your performance, GOAL Score** produces a reliable number that will give you confidence in your preparation before you sit for your exam.



If your GOAL Score is a 70 or above, you are well-prepared to sit for your exam!

62	74	78
General Probability	Random Variables with Univariate Probability Distributions	Random Variables with Univariate Probability Distributions
Weighting: 13.4%	Weighting: 43.3%	Weighting: 43.3%

See key areas where you can improve.

General Probability — Set functions including set notation and basic elements of probability

Difficulty	Mean	Proportion
Core	58	92 / 304
Advanced	60	169 / 304
Mastery	78	43 / 304

Detailed performance tracking.

GOAL Session Activity Summary — Filter: All Types, All Statuses

Created	Last Accessed	Completed	Mode	Categories	Questions	Status	Status
05/24/2022 21:57:43	05/24/2022 21:57:43		Quiz	Continuous P...	25	New	Resume
05/24/2022 10:34:05	05/24/2022 14:57:49	05/24/2022 14:57:49	Practice Session	Addition and ...	80	Complete	Review
05/21/2022 19:32:50	05/23/2022 20:02:00		Simulated Exam	Exam 2	30	Reviewing	Complete
05/17/2022 15:19:19	05/17/2022 15:46:03	05/23/2022 14:15:29	Simulated Exam	Exam 6	30	Complete	Review
05/14/2022 11:26:59	05/14/2020 12:02:36	05/23/2022 11:57:47	Quiz	Conditional D...	20	Complete	Review

Quickly return to previous sessions.

Contents

Preface

Exam FAM-S discusses the mathematics of short-term insurance. Short-term insurance is insurance for periods of one year or less. For these lines of insurance, there is no guaranteed renewal, and even if the insurance is renewed from year to year, premium rates are usually updated every year. Also, there is no cost for switching insurers from one year to the next. This is unlike long-term insurance such as life insurance.

As a candidate for ASA and FSA, you probably won't be working in a property/casualty insurance company. And the typical insurance discussed in this course is auto insurance. But some of the methods we discuss are useful for medical and dental insurance, products sold by life insurance companies. And some concepts, such as credibility, are useful for mortality studies and reserving.

Prerequisites for most of the material are few beyond knowing probability (and calculus of course). Some elementary statistics will be helpful, especially for Part IV of the manual.

This manual

The exercises in this manual

I've provided lots of my own exercises, as well as relevant exercises from old exams. Though the style of exam questions has changed a little, these are still very useful practice exercises which cover the same material—don't dismiss them as obsolete!

All SOA or joint exam questions in this manual from exams given in 2000 and later, with solutions, are also available on the web from the SOA. When the 2000 syllabus was established in 1999, sample exams 3 and 4 were created, consisting partially of questions from older exams and partially of new questions, not all multiple choice. These sample exams were not real exams, and some questions were inappropriate or defective. These sample exams are no longer posted on the web. I have included appropriate questions, labeled "1999 C3 Sample" or "1999 C4 Sample". *These refer to these 1999 sample exams, not to more recent sets of sample questions that may be posted.*

Questions from old exams are marked xxx:yy, where xxx is the time the exam was given, with S for spring and F for fall followed by a 2-digit year, and yy is the question number. Sometimes xxx is preceded with SOA or CAS to indicate the sponsoring organization. From about 1986 to 2000, SOA exams had 3-digit numbers (like 160) and CAS exams were a number and a letter (like 4B). From 2000 to Spring 2003, exam 3 was jointly sponsored, so I do not indicate "SOA" or "CAS" for exam 3 questions from that period. There was a period in the 1990's when the SOA, while releasing old exam questions, did not indicate which exam they came from. As a result, I sometimes cannot identify the source exam for questions from this period. In such a case, I mark the question aaa-bb-cc:yy, where aaa-bb-cc is the study note number and yy is the question number. Generally aaa is the exam number (like 160), and cc is the 2-digit year the study note was published.

Index

This manual has an index. Whenever you remember some topic in this manual but can't remember where you saw it, check the index. If it isn't in the index but you're sure it's in the manual and an index listing would be appropriate, contact the author.

Flashcards

Many students find flashcards a useful tool for learning key formulas and concepts. ASM flashcards, available from the same distributors that sell this manual, contain the formulas and concepts from this manual in a convenient deck of cards. The cards have crossreferences, usually by page, to the manual.

Downloads from the SOA website

Tables

At the exam, you will be given distribution tables. They are available at the following link:

https://www.soa.org/49f99d/globalassets/assets/files/edu/2022/tables-fam-s.pdf

These give you moments and distribution functions for many distributions, so you need not memorize them. It also gives you a chi-square distribution table, which will have no use on this exam. (It would be useful on ASTAM, but they don't give it to you for that exam—they instead give you a worksheet that includes the chi-square function.)

One of the things that is missing from it, that used to be provided, is the normal distribution table. They will give you the Prometric calculator, which will calculate the standard normal distribution and its inverse to 5 decimal places. This precision is especially useful for option pricing, but doesn't make much difference in the rest of this course. While this manual usually works out the exercises with precision, I've used the traditional 3-digit approximations of the 90^{th}, 95^{th}, and 97.5^{th} percentiles (1.282, 1.645, and 1.960 respectively) in my solutions.

Notation and terminology note

The notation and terminology note are at

https://www.soa.org/4a1b9f/globalassets/assets/files/edu/2022/fam-s-notation-note.pdf

This manual uses notation and terminology in accordance with that note.

Sample questions and solutions

The following links have 76 sample questions and solutions for this exam:

https://www.soa.org/49f968/globalassets/assets/files/edu/2022/fam-s-sample-questions.pdf
https://www.soa.org/49f992/globalassets/assets/files/edu/2022/fam-s-sample-solutions.pdf

To appreciate these questions, let's go through their history. During 2000–Spring 2007, the SOA released associateship exams 9 times. That was the pre-CBT era. Later on, the SOA compiled a list of about 300 sample questions based on 6 of those exams; they did not use the two 2000 administrations and did not use the Spring 2007 exam. The syllabus of C did not change much; a few questions were deleted as their topics dropped from the syllabus, but nothing was added for the few new topics. In 2018, when Exam C became Exam STAM, insurance coverages were added to the syllabus, and 22 additional sample questions were added.

The 76 sample questions you have were selected from the old list; nothing was added. They did not include every question that was relevant. But strangely—probably by error—they included 6 questions not covered by the syllabus reading, questions on topics assigned to ASTAM. Refer to Table B.2, which has and "NS" indicator for those questions. They edited the questions slightly, but no material changes were made. This manual does not include those 6 questions, but does include the others, as well as other old exam questions not in that list.

As a result of the way the list was compiled, the number of questions on each topic is not in accordance with syllabus weights. More importantly, no questions are provided on topics added to FAM-S that were not on STAM, namely option pricing.

Notes about the exam

Topic weights

The syllabus breaks the course down into 6 topics. Their weights[1] and the lessons in the manual that cover them are:

[1]The weights listed in the syllabus are for the full FAM exam. I've doubled the weights so that they represent the weights for FAM-S.

Table 1: Nine Week Study Schedule for Exam STAM

Week	Subject	Lessons	Rarely Tested
1	Probability basics	1–4	2.3, 2.4
2	Short term insurances and loss reserves	5–7	
3	Ratemaking	8–9	
4	Severity modifications	10–12	
5	Reinsurance, risk measures, and frequency	13–18	14.4, 14.5, *16*, 17.2
6	Aggregate loss	19–22	*19.3*
7	Maximum likelihood	23–25	
8	Credibility	26–28	27
9	Option Pricing	29–31	

Topic	Weight	Lessons
1. Insurance and Reinsurance Coverages	15–25%	5–6, 10–13, 15–16
2. Severity, Frequency, and Aggregate Models	25–30%	1–4, 14, 17–22
3. Parametric and Non-Parametric Estimation	10–20%	23–25
4. Introduction to Credibility	5–10%	26–28
5. Pricing and Reserving for Short-Term Insurance Coverages	15–25%	7–9
6. Option Pricing Fundamentals	5–15%	29–31

Since the exam has 20 questions, each 5% is one question.

Guessing penalty

There is no guessing penalty on this exam. So fill in every answer—you may be lucky! Leave yourself a couple of seconds to do this.

Calculators

A wide variety of calculators are permitted: the TI-30Xa, TI-30X II battery or solar, TI-30X MultiView battery or solar, the BA-35 (battery or solar), and the BA-II Plus (or BA II Plus Professional Edition). You may bring several calculators into the exam. The MultiView calculator is considered the best one, due to its data tables which allow fast statistical calculations. The data table is a very restricted spreadsheet. Despite its limitations, it is useful.

Another feature of the Multiview is storage of previous calculations. They can be recalled and edited.

Other features which may be of use are the K constant and the table feature, which allows calculation of a function at selected values or at values in an arithmetic progression.

Financial calculations do not occur on this exam; interest is almost never considered. You will not miss the lack of financial functions on the Multiview.

Study Schedule

Different students will have different speeds and different constraints, so it's hard to create a study schedule useful for everybody. However, I offer a sample 9-week study schedule, Table 1, as a guide. The last column lists rarely tested materials so you can skip those if you are behind in your schedule. Italicized sections in this column are, in my opinion, extremely unlikely exam topics.

Errata

Please report any errors you find. Reports may be sent to the publisher (mail@studymanuals.com) or directly to me (errata@aceyourexams.net). *When reporting errata, please indicate which manual and which edition and printing you are referring to!* This manual is the 1st edition 1st printing of the Exam FAM-S manual.

An errata list will be posted at http://errata.aceyourexams.net

Acknowledgements

I wish to thank the Society of Actuaries and the Casualty Actuarial Society for permission to use their old exam questions. These questions are the backbone of this manual.

I wish to thank Donald Knuth, the creator of TEX, Leslie Lamport, the creator of LATEX, and the many package writers and maintainers, for providing a typesetting system that allows such beautiful typesetting of mathematics and figures. I hope you agree, after looking at mathematical material apparently typed with Word (e.g., the Dean study note) that there's no comparison in appearance.

Part I

Probability and Insurance Coverages

Lesson 2

Parametric Distributions

Reading: *Loss Models* Fifth Edition 4, 5.3—5.4

A ***parametric distribution*** is one that is defined by a fixed number of parameters. Examples of parametric distributions are the **exponential distribution** (parameter θ) and the **Pareto distribution** (parameters α, θ). Any distribution listed in the *Loss Models* appendix is parametric.

The alternative to a parametric distribution is a ***data-dependent distribution***. A data-dependent distribution is one where the specification requires at least as many "parameters" as the number of data points in the sample used to create it; the bigger the sample, the more "parameters".

It is traditional to use parametric distributions for claim counts (**frequency**) and loss size (**severity**). Parametric distributions have many advantages. One of the advantages of parametric distributions which makes them so useful for severity is that they handle inflation easily.

2.1 Scaling

A parametric distribution is a member of a ***scale family*** if any positive multiple of the random variable has the same form. In other words, the distribution function of cX, for c a positive constant, is of the same form as the distribution function of X, but with different values for the parameters. Sometimes the distribution can be parametrized in such a way that only one parameter of cX has a value different from the parameters of X. If the distribution is parametrized in this fashion, so that the only parameter of cX having a different value from X is θ, and the value of θ for cX is c times the value of θ for X, then θ is called a ***scale parameter***.

All of the continuous distributions in the tables (Appendix A) are scale families. The parametrizations given in the tables are often different from those you would find in other sources, such as your probability textbook. They are parametrized so that θ is the scale parameter. Thus when you are given that a random variable has any distribution in the appendix and you are given the parameters, it is easy to determine the distribution of a multiple of the random variable.

The only distributions not parametrized with a scale parameter are the **lognormal** and the **inverse Gaussian**. Even though the inverse Gaussian has θ as a parameter, it is not a scale parameter. The parametrization for the lognormal given in the tables is the traditional one. *If you need to **scale a lognormal**, proceed as follows: if X is lognormal with parameters (μ, σ), then cX is lognormal with parameters $(\mu + \ln c, \sigma)$.*

To scale a random variable not in the tables, you'd reason as follows. Let $Y = cX$, $c > 0$. Then

$$F_Y(y) = \Pr(Y \le y) = \Pr(cX \le y) = \Pr\left(X \le \frac{y}{c}\right) = F_X\left(\frac{y}{c}\right)$$

One use of scaling is in handling inflation. In fact, handling inflation is the only topic in this lesson that is commonly tested directly. If loss sizes are inflated by $100r\%$, the **inflated loss variable** Y will be $(1 + r)X$, where X is the pre-inflation loss variable. For a scale family with a scale parameter, you just multiply θ by $(1 + r)$ to obtain the new distribution.

Example 2A Claim sizes expressed in dollars follow a two-parameter Pareto distribution with parameters $\alpha = 5$ and $\theta = 90$. A euro is worth \$1.50.

Calculate the probability that a claim will be for 20 euros or less. ■

Solution: If claim sizes in dollars are X, then claim sizes in euros are $Y = X/1.5$. The resulting euro-based random variable Y for claim size will be Pareto with $\alpha = 5$, $\theta = 90/1.5 = 60$. The probability that a claim will be no more than 20 euros is

$$\Pr(Y \le 20) = F_Y(20) = 1 - \left(\frac{60}{60+20}\right)^5 = \boxed{0.7627}$$

□

Example 2B Claim sizes in 2020 follow a lognormal distribution with parameters $\mu = 4.5$ and $\sigma = 2$. Claim sizes grow at 6% uniform inflation during 2021 and 2022.

Calculate $f(1000)$, the probability density function at 1000, of the claim size distribution in 2022. ■

Solution: If X is the claim size random variable in 2020, then $Y = 1.06^2X$ is the revised variable in 2022. The revised lognormal distribution of Y has parameters $\mu = 4.5 + 2\ln 1.06$ and $\sigma = 2$. The probability density function at 1000 is

$$\begin{aligned} f_Y(1000) &= \frac{1}{\sigma(1000)\sqrt{2\pi}}e^{-(\ln 1000-\mu)^2/2\sigma^2} \\ &= \frac{1}{(2)(1000)\sqrt{2\pi}}e^{-[\ln 1000-(4.5+2\ln 1.06)]^2/2(2^2)} \\ &= (0.000199471)(0.518814) = \boxed{0.0001035} \end{aligned}$$

□

Example 2C Claim sizes expressed in dollars follow a lognormal distribution with parameters $\mu = 3$ and $\sigma = 2$. A euro is worth \$1.50.

Calculate the probability that a claim will be for 100 euros or less. ■

Solution: If claim sizes in dollars are X, then claim sizes in euros are $Y = X/1.5$. As discussed above, the distribution of claim sizes in euros is lognormal with parameters $\mu = 3 - \ln 1.5$ and $\sigma = 2$. Then

$$F_Y(100) = \Phi\left(\frac{\ln 100 - 3 + \ln 1.5}{2}\right) = \Phi(1.01) = \boxed{0.8438}$$

□

Example 2D Claim sizes X initially follow a distribution with distribution function:

$$F_X(x) = 1 - \frac{1}{e^{0.01x}(1+0.01x)} \qquad x > 0$$

Claim sizes are inflated by 50% uniformly.

Calculate the probability that a claim will be for 60 or less after inflation. ■

Solution: Let Y be the increased claim size. Then $Y = 1.5X$, so $\Pr(Y \le 60) = \Pr(X \le 60/1.5) = F_X(40)$.

$$F_X(40) = 1 - \frac{1}{1.4e^{0.4}} = \boxed{0.5212}$$

□

2.2 Common parametric distributions

The tables provide a lot of information about the distributions, but if you don't recognize the distribution, you won't know to use the table. Therefore, it is a good idea to be familiar with the common distributions.

You should familiarize yourself with the *form* of each distribution, but not necessarily the constants. The constant is forced so that the density function will integrate to 1. If you know which distribution you are dealing with, you can figure out the constant. To emphasize this point, in the following discussion, we will use the letter c for constants rather than spelling out what the constants are. You are not trying to recognize the constant; you are trying to recognize the form.

We will mention the means and variances or second moments of the distributions. You need not memorize any of these. The tables give you the **raw moments**. You can calculate the **variance** as $\mathbf{E}[X^2] - \mathbf{E}[X]^2$. However, for

The gamma function

The gamma function $\Gamma(x)$ is a generalization to real numbers of the factorial function, defined by

$$\Gamma(x) = \int_0^\infty u^{x-1} e^{-u} \mathrm{d}u$$

For positive integers n,

$$\Gamma(n) = (n-1)!$$

The most important relationship for $\Gamma(x)$ that you should know is

$$\Gamma(x+1) = x\Gamma(x)$$

for any real number x.

EXAMPLE 2E Evaluate $\dfrac{\Gamma(8.5)}{\Gamma(6.5)}$.

SOLUTION:

$$\frac{\Gamma(8.5)}{\Gamma(6.5)} = \left(\frac{\Gamma(8.5)}{\Gamma(7.5)}\right)\left(\frac{\Gamma(7.5)}{\Gamma(6.5)}\right) = (7.5)(6.5) = \boxed{\mathbf{48.75}}$$

---

frequently used distributions, you may want to memorize the mean and variance to save yourself some time when working out questions.

We will graph the distributions. You are not responsible for graphs, but they may help you understand the distributions.

The tables occasionally use the **gamma function** $\Gamma(x)$ in the formulas for the moments. You should have a basic knowledge of the gamma function; if you are not familiar with this function, see the sidebar. The tables also use the **incomplete gamma** and **beta functions**, and define them, but you can get by without knowing them.

2.2.1 Uniform

A **uniform distribution** has a constant density on $[d, u]$:

$$f(x; d, u) = \frac{1}{u-d} \qquad d \le x \le u$$

$$F(x; d, u) = \begin{cases} 0 & x \le d \\ \dfrac{x-d}{u-d} & d \le x \le u \\ 1 & x \ge u \end{cases}$$

You recognize a uniform distribution both by its finite **support**[1] and by the lack of an x in the density function.

Its moments are

$$\mathbf{E}[X] = \frac{d+u}{2}$$

$$\mathrm{Var}(X) = \frac{(u-d)^2}{12}$$

[1]"Support" is the range of values for which the probability density function is nonzero.

Its mean, median, and midrange are equal. The best way to calculate the second moment is to add up the variance and the square of the mean. However, some students prefer to use the following easy-to-derive formula:

$$\mathbf{E}[X^2] = \frac{1}{u-d}\int_d^u x^2\,\mathrm{d}x = \frac{u^3-d^3}{3(u-d)} = \frac{u^2+ud+d^2}{3} \tag{2.1}$$

If $d = 0$, then the formula reduces to $u^2/3$.

The uniform distribution is not directly in the tables, so I recommend you memorize the formulas for mean and variance. However, if $d = 0$, then the uniform distribution is a special case of a beta distribution with $\theta = u$, $a = 1$, $b = 1$.

2.2.2 Beta

The probability density function of a **beta distribution** with $\theta = 1$ has the form

$$f(x; a, b) = cx^{a-1}(1-x)^{b-1} \qquad 0 \le x \le 1$$

The parameters a and b must be positive. They may equal 1, in which case the corresponding factor is missing from the density function. Thus if $a = b = 1$, the beta distribution is a uniform distribution.

You recognize a beta distribution both by its finite support—it's the only common distribution for which the density is nonzero only on a finite range of values—and by factors with x and $1-x$ raised to powers and no other use of x in the density function.

If θ is arbitrary, then the form of the probability density function is

$$f(x; a, b, \theta) = cx^{a-1}(\theta-x)^{b-1} \qquad 0 \le x \le \theta$$

The distribution function can be evaluated if a or b is an integer. The moments are

$$\mathbf{E}[X] = \frac{\theta a}{a+b}$$
$$\mathrm{Var}(X) = \frac{\theta^2 ab}{(a+b)^2(a+b+1)}$$

The mode is $\theta(a-1)/(a+b-2)$ when a and b are both greater than 1, but you are not responsible for this fact.

Figure 2.1 graphs four beta distributions with $\theta = 1$ all having mean $2/3$. You can see how the distribution becomes more peaked and normal looking as a and b increase.

2.2.3 Exponential

The probability density function of an **exponential distribution** has the form

$$f(x; \theta) = ce^{-x/\theta} \qquad x \ge 0$$

θ must be positive.

You recognize an exponential distribution when the density function has e raised to a multiple of x, and no other use of x.

The distribution function is easily evaluated. The moments are:

$$\mathbf{E}[X] = \theta$$
$$\mathrm{Var}(X) = \theta^2$$

Figure 2.2 graphs three exponential distributions. The higher the parameter, the more weight placed on higher numbers.

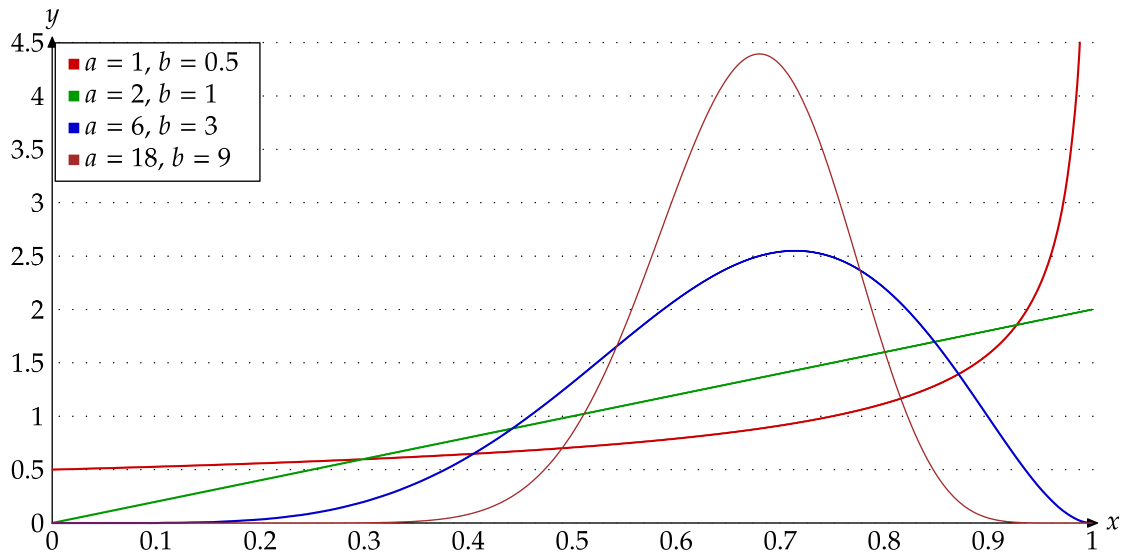


Figure 2.1: Probability density function of four beta distributions with $\theta = 1$ and mean 2/3

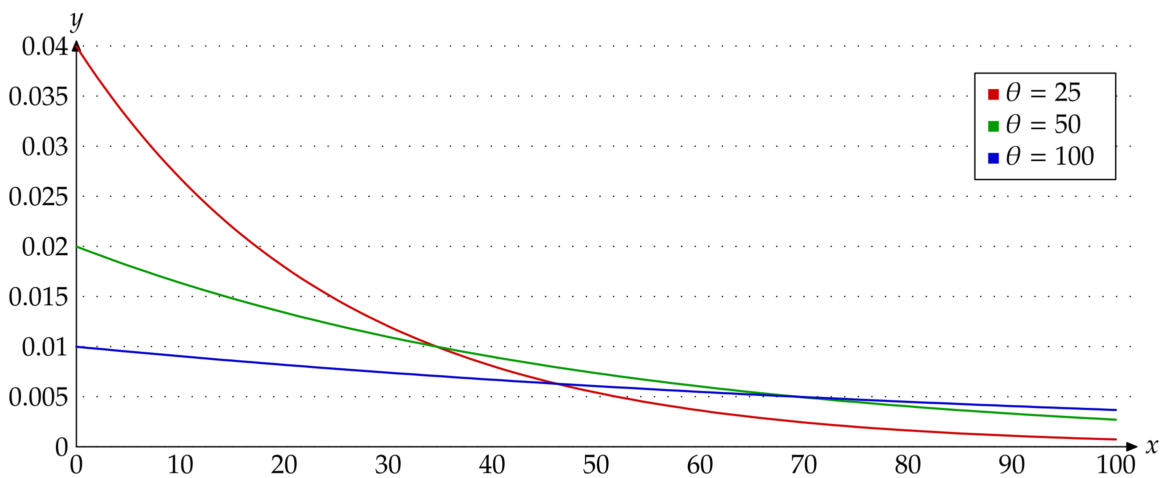


Figure 2.2: Probability density function of three exponential distributions

2.2.4 Weibull

A **Weibull distribution** is a transformed exponential distribution. If Y is exponential with mean μ, then $X = Y^{1/\tau}$ is Weibull with parameters $\theta = \mu^{1/\tau}$ and τ. An exponential is a special case of a Weibull with $\tau = 1$.

The form of the density function is

$$f(x;\tau,\theta) = cx^{\tau-1}e^{-(x/\theta)^\tau} \qquad x \geq 0$$

Both parameters must be positive.

You recognize a Weibull distribution when the density function has e raised to a multiple of a power of x, and in addition has a corresponding power of x, one lower than the power in the exponential, as a factor.

The distribution function is easily evaluated, but the moments require evaluating the **gamma function**, which usually requires numerical techniques. The moments are

$$\mathbf{E}[X] = \theta\Gamma(1 + 1/\tau)$$
$$\mathbf{E}[X^2] = \theta^2\Gamma(1 + 2/\tau)$$

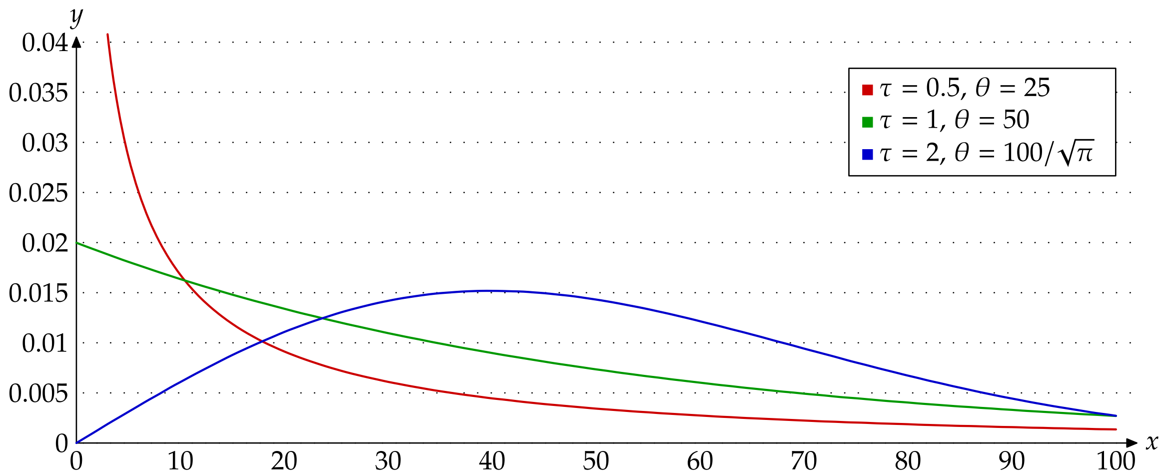


Figure 2.3: Probability density function of three Weibull distributions with mean 50

Figure 2.3 graphs three Weibull distributions with mean 50. The distribution has a non-zero mode when $\tau > 1$. Notice that the distribution with $\tau = 0.5$ puts a lot of weight on small numbers. To make up for this, it will also have to put higher weight than the other two distributions on very large numbers, so although it's not shown, its graph will cross the other two graphs for high x

2.2.5 Gamma

The form of the density function of a **gamma distribution** is

$$f(x;\alpha,\theta) = cx^{\alpha-1}e^{-x/\theta} \qquad x \ge 0$$

Both parameters must be positive.

When α is an integer, a gamma random variable with parameters α and θ is the sum of α independent exponential random variables with parameter θ. In particular, when $\alpha = 1$, the gamma random variable is exponential. The gamma distribution is called an **Erlang distribution** when α is an integer.

You recognize a gamma distribution when the density function has e raised to a multiple of x, and in addition has x raised to a power. Contrast this with a Weibull, where e is raised to a multiple of a *power* of x.

The distribution function may be evaluated if α is an integer; otherwise numerical techniques are needed. However, the moments are easily evaluated:

$$\mathbf{E}[X] = \alpha\theta$$
$$\mathrm{Var}(X) = \alpha\theta^2$$

Figure 2.4 graphs three gamma distributions with mean 50. As α goes to infinity, the graph's peak narrows and the distribution converges to a normal distribution.

The gamma distribution is one of the few for which the moment generating function has a closed form. In particular, the moment generating function of an exponential has a closed form. The only other distributions in the tables with closed form moment generating functions are the normal distribution (not actually in the tables, but the formula for the lognormal moments is the MGF of a normal) and the inverse Gaussian.

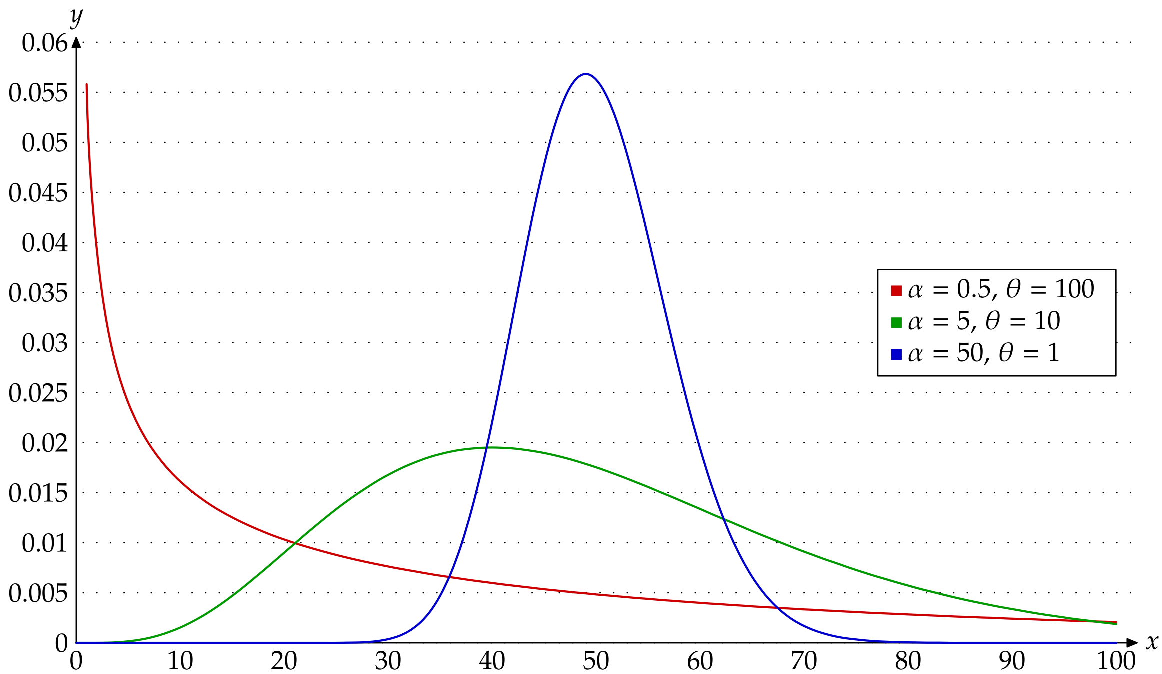


Figure 2.4: Probability density function of three gamma distributions with mean 50

2.2.6 Pareto

When we say "Pareto", we mean a *two-parameter* Pareto. On recent exams, they write out "two-parameter" to make it clear, but on older exams, you will often find the word "Pareto" with no qualifier. It always refers to a two-parameter Pareto, not a single-parameter Pareto.

The form of the density function of a **two-parameter Pareto** is

$$f(x) = \frac{c}{(\theta + x)^{\alpha+1}} \qquad x \geq 0$$

Both parameters must be positive.

You recognize a Pareto when the density function has a denominator with x plus a constant raised to a power.

The distribution function is easily evaluated. The moments are

$$\mathbf{E}[X] = \frac{\theta}{\alpha - 1} \qquad \alpha > 1$$

$$\mathbf{E}[X^2] = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} \qquad \alpha > 2$$

When α does not satisfy these conditions, the corresponding moments don't exist.

A shortcut formula for the variance of a Pareto is

$$\mathrm{Var}(X) = \mathbf{E}[X]^2\left(\frac{\alpha}{\alpha - 2}\right)$$

Figure 2.5 graphs three Pareto distributions, one with $\alpha < 1$ and the other two with mean 50. Although the one with $\alpha = 0.5$ puts higher weight on small numbers than the other two, its mean is infinite; it puts higher weight on large numbers than the other two, and its graph eventually crosses the other two as $x \to \infty$.

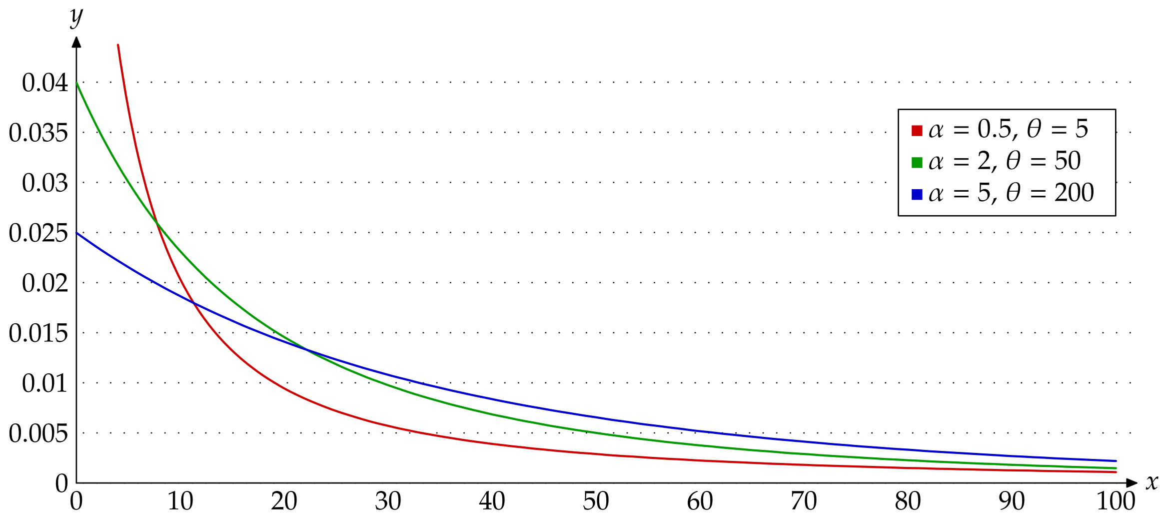


Figure 2.5: Probability density function of three Pareto distributions

2.2.7 Single-parameter Pareto

The form of the density function of a **single-parameter Pareto** is

$$f(x) = \frac{c}{x^{\alpha+1}} \qquad x \geq \theta$$

α must be positive. θ is not considered a parameter since it must be selected in advance, based on what you want the range to be.

You recognize a single-parameter Pareto by the range of nonzero values for its density function—unlike most other distributions, this range does not start at 0—and by the form of the density function, which has a denominator with x raised to a power. A beta distribution may also have x raised to a negative power, but its density function is 0 above a finite number.

A single-parameter Pareto X is a two-parameter Pareto Y shifted by θ: $X = Y + \theta$. Thus it has the same variance, and the mean is θ greater than the mean of a two-parameter Pareto with the same parameters.

$$\mathbf{E}[X] = \frac{\alpha\theta}{\alpha - 1} \qquad \alpha > 1$$

$$\mathbf{E}[X^2] = \frac{\alpha\theta^2}{\alpha - 2} \qquad \alpha > 2$$

2.2.8 Lognormal

The form of the density function of a **lognormal distribution** is

$$f(x) = \frac{ce^{-(\ln x - \mu)^2/2\sigma^2}}{x} \qquad x > 0$$

σ must be nonnegative.

You recognize a lognormal by the $\ln x$ in the exponent.

If Y is normal, then $X = e^Y$ is lognormal with the same parameters μ and σ. Thus, to calculate the distribution function, use

$$F_X(x) = F_Y(\ln x) = \Phi\left(\frac{\ln x - \mu}{\sigma}\right)$$

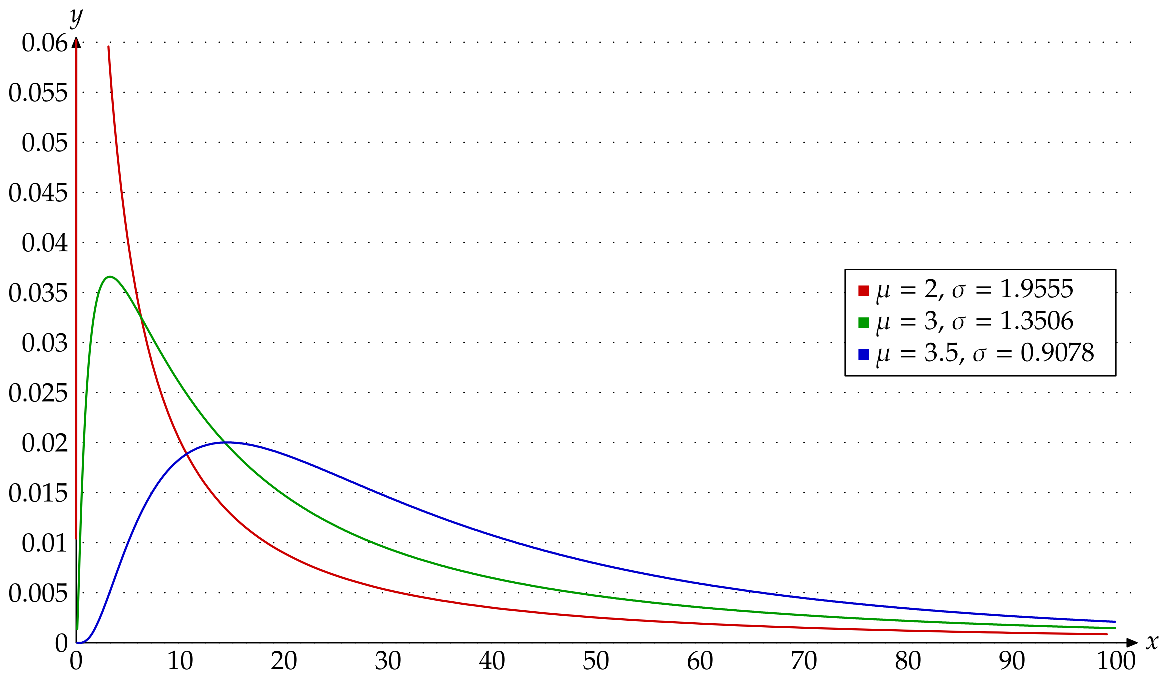


Figure 2.6: Probability density function of three lognormal distributions with mean 50

where $\Phi(x)$ is the standard normal distribution function, for which you are given tables. The moments of a lognormal are

$$\mathbf{E}[X] = e^{\mu+0.5\sigma^2}$$
$$\mathbf{E}[X^2] = e^{2\mu+2\sigma^2}$$

More generally, $\mathbf{E}[X^k] = \mathbf{E}[e^{kY}] = M_Y(k)$, where $M_Y(k)$ is the moment generating function of the corresponding normal distribution.

Figure 2.6 graphs three lognormals with mean 50. The mode is $\exp(\mu - \sigma^2)$, as stated in the tables. For $\mu = 2$, the mode is off the graph. As σ gets lower, the distribution flattens out.

Table 2.1 is a summary of the forms of probability density functions for common distributions.

2.3 The linear exponential family

The following material is based on *Loss Models* 5.4 which is on the syllabus. It won't play any role in this course, but the **linear exponential family** is commonly used for generalized linear models[2], which you'll study when working on Exam SRM. I doubt anything in this section will be tested on directly, so you may skip it.

A set of parametric distributions is in the linear exponential family if it can be parametrized with a parameter θ in such a way that in its density function, the only interaction between θ and x is in the exponent of e, which is x times a function of θ. In other words, its density function $f(x;\theta)$ can be expressed as

$$f(x;\theta) = \frac{p(x)e^{r(\theta)x}}{q(\theta)}$$

The set may have other parameters. $q(\theta)$ is the normalizing constant which makes the integral of f equal to 1. $r(\theta)$ is called the *canonical parameter* of the distribution.

[2]Many textbooks leave out "linear" and just call it the exponential family.

Table 2.1: Forms of probability density functions for common distributions

Distribution	Probability density function	
Uniform	c	$d \le x \le u$
Beta	$cx^{a-1}(\theta - x)^{b-1}$	$0 \le x \le \theta$
Exponential	$ce^{-x/\theta}$	$x \ge 0$
Weibull	$cx^{\tau-1}e^{-x^\tau/\theta^\tau}$	$x \ge 0$
Gamma	$cx^{\alpha-1}e^{-x/\theta}$	$x \ge 0$
Pareto	$\dfrac{c}{(x+\theta)^{\alpha+1}}$	$x \ge 0$
Single-parameter Pareto	$\dfrac{c}{x^{\alpha+1}}$	$x \ge \theta$
Lognormal	$\dfrac{ce^{-(\ln x-\mu)^2/2\sigma^2}}{x}$	$x > 0$

Examples of the linear exponential family are:

Gamma distribution The pdf is

$$f(x;\mu,\sigma) = \frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha}$$

Let $r(\theta) = -1/\theta$, $p(x) = x^{\alpha-1}$, and $q(\theta) = \Gamma(\alpha)\theta^\alpha$.

Normal distribution The pdf is

$$f(x;\theta) = \frac{e^{-(x-\mu)^2/2\sigma^2}}{\sigma\sqrt{2\pi}}$$

Let $\theta = \mu$. The denominator of the pdf does not have x or θ so it can go into $q(\theta)$ or into $p(x)$. The exponent can be expanded into

$$-\frac{x^2}{2\sigma^2} + \frac{x\theta}{\sigma^2} - \frac{\theta^2}{2\sigma^2}$$

and only the second summand involves both x and θ, and x appears to the first power. Thus we can set $p(x) = e^{-x^2/2\sigma^2}$, $r(\theta) = \theta/\sigma^2$, and $q(\theta) = e^{\theta^2/2\sigma^2}\sigma\sqrt{2\pi}$.

Discrete distributions are in the linear exponential family if we can express the probability function in the linear exponential form.

Poisson distribution For a Poisson distribution, the probability function is

$$f(x;\lambda) = e^{-\lambda}\frac{\lambda^x}{x!} = e^{-\lambda}\frac{e^{x\ln\lambda}}{x!}$$

We can let $\theta = \lambda$, and then $p(x) = 1/x!$, $r(\theta) = \ln\theta$, and $q(\theta) = e^\theta$.

The textbook develops the following formulas for the mean and variance of a distribution from the linear exponential family:

$$\begin{aligned}\mathbf{E}[X] = \mu(\theta) &= \frac{q'(\theta)}{r'(\theta)q(\theta)} = \frac{\big(\ln q(\theta)\big)'}{r'(\theta)}\\ \operatorname{Var}(X) = v(\theta) &= \frac{\mu'(\theta)}{r'(\theta)}\end{aligned}$$

Thus, in the above examples:

Gamma distribution

$$\begin{aligned}\frac{\mathrm{d}\ln q}{\mathrm{d}\theta} &= \frac{\alpha}{\theta}\\ \frac{\mathrm{d}r}{\mathrm{d}\theta} &= \frac{1}{\theta^2}\\ \mathbf{E}[X] &= \frac{\alpha/\theta}{1/\theta^2} = \alpha\theta\\ \operatorname{Var}(X) &= \frac{\alpha}{1/\theta^2} = \alpha\theta^2\end{aligned}$$

Normal distribution

$$\begin{aligned}\big(\ln q(\theta)\big)' &= \frac{2\theta}{2\sigma^2} = \frac{\theta}{\sigma^2}\\ r'(\theta) &= \frac{1}{\sigma^2}\\ \mathbf{E}[X] &= \frac{\theta/\sigma^2}{1/\sigma^2} = \theta\\ \operatorname{Var}(X) &= \frac{1}{1/\sigma^2} = \sigma^2\end{aligned}$$

Poisson distribution

$$\begin{aligned}\big(\ln q(\theta)\big)' &= 1\\ r'(\theta) &= \frac{1}{\theta}\\ \mathbf{E}[X] &= \frac{1}{1/\theta} = \theta\\ \operatorname{Var}(X) &= \frac{1}{1/\theta} = \theta\end{aligned}$$

2.4 Limiting distributions

The following material is based on *Loss Models* 5.3.3. I don't think it has ever appeared on the exam and doubt it ever will.

In some cases, as the parameters of a distribution go to infinity, the distribution converges to another distribution. To demonstrate this, we will usually have to use the identity

$$\lim_{\alpha\to\infty}\left(1+\frac{r}{\alpha}\right)^{\alpha} = e^r$$

Equivalently, if c is a constant (not dependent on α), then

$$\lim_{\alpha\to\infty}\left(1+\frac{r}{\alpha}\right)^{\alpha+c}=\lim_{\alpha\to\infty}\left(1+\frac{r}{\alpha}\right)^{\alpha}\left(1+\frac{r}{\alpha}\right)^{c}=e^{r}$$

As a simple example (not in the textbook) of a limiting distribution, consider a **gamma distribution** with a fixed mean μ, and let $\alpha\to\infty$. Then $\theta=\mu/\alpha$. The **moment generating function** is

$$M(t)=(1-\theta t)^{-\alpha}=\frac{1}{\left(1-\frac{\mu t}{\alpha}\right)^{\alpha}}$$

and as $\alpha\to\infty$, the denominator goes to $e^{-\mu t}$, so $M(t)\to e^{\mu t}$, which is the moment generating function of the constant μ. So as $\alpha\to\infty$, the limiting distribution of a gamma is a distribution equal to the mean with probability 1.

As another example, let's carry out textbook exercise 5.21, which asks you to demonstrate that the limiting distribution of a **Pareto** with θ/α constant as $\alpha\to\infty$ is an exponential. Let $k=\theta/\alpha$. The density function of a Pareto is

$$\begin{aligned}f(x;\alpha,\theta)&=\frac{\alpha\theta^{\alpha}}{(\theta+x)^{\alpha+1}}=\frac{\alpha(\alpha k)^{\alpha}}{(\alpha k+x)^{\alpha+1}}\\&=\frac{k^{\alpha}}{(k+x/\alpha)^{\alpha+1}}=\frac{1}{k\big(1+(x/k)/\alpha\big)^{\alpha+1}}\end{aligned}$$

and the limit as $\alpha\to\infty$ is $(1/k)e^{-x/k}$. That is the density function of an exponential with mean k. Notice that as $\alpha\to\infty$, the mean of the Pareto converges to k.

Table 2.2: Summary of Parametric Distribution Concepts

- If X is a member of a scale family with scale parameter θ with value s, then cX is in the same family and has the same parameter values as X except that the scale parameter θ has value cs.
- All distributions in the tables are scale families with scale parameter θ except for lognormal and inverse Gaussian.
- If X is lognormal with parameters μ and σ, then cX is lognormal with parameters $\mu+\ln c$ and σ.
- If X is normal with parameters μ and σ^2, then e^X is lognormal with parameters μ and σ.
- See Table 2.1 to learn the forms of commonly occurring distributions. Useful facts are

Uniform on [d,u]	$\mathbf{E}[X]=\dfrac{d+u}{2}$
	$\mathrm{Var}(X)=\dfrac{(u-d)^2}{12}$
Uniform on [0,u]	$\mathbf{E}[X^2]=\dfrac{u^2}{3}$
Gamma	$\mathrm{Var}(X)=\alpha\theta^2$

- If Y is single-parameter Pareto with parameters α and θ, then $Y-\theta$ is two-parameter Pareto with the same parameters.
- X is in the linear exponential family if its probability density function can be expressed as

$$f(x;\theta)=\frac{p(x)e^{r(\theta)x}}{q(\theta)}$$

Exercises

2.1. Loss sizes for an insurance coverage follow an inverse gamma distribution with mean 6 and mode 4.
Calculate the coefficient of skewness for the losses.

(A) 3.1 (B) 3.2 (C) 3.3 (D) 3.4 (E) 3.5

2.2. For a commercial fire coverage

- In 2021, loss sizes follow a two-parameter Pareto distribution with parameters $\alpha = 4$ and θ.
- In 2022, there is uniform inflation at rate r.
- The 65^{th} percentile of loss size in 2022 equals the mean loss size in 2021.

Determine r.

2.3. [**CAS3-S06:26**] The aggregate losses of Eiffel Auto Insurance are denoted in euro currency and follow a lognormal distribution with $\mu = 8$ and $\sigma = 2$.

Given that 1 euro = 1.3 dollars, which set of lognormal parameters describes the distribution of Eiffel's losses in dollars?

(A) $\mu = 6.15$, $\sigma = 2.26$
(B) $\mu = 7.74$, $\sigma = 2.00$
(C) $\mu = 8.00$, $\sigma = 2.60$
(D) $\mu = 8.26$, $\sigma = 2.00$
(E) $\mu = 10.40$, $\sigma = 2.60$

2.4. [**4B-S90:37**] (2 points) Liability claim severity follows a Pareto distribution with a mean of 25,000 and parameter $\alpha = 3$.

If inflation increases all claims by 20%, the probability of a claim exceeding 100,000 increases by what amount?

(A) Less than 0.02
(B) At least 0.02, but less than 0.03
(C) At least 0.03, but less than 0.04
(D) At least 0.04, but less than 0.05
(E) At least 0.05

Exercises continue on the next page . . .

2.5. **[4B-F97:26]** (3 points) You are given the following:

- In 1996, losses follow a lognormal distribution with parameters μ and σ.
- In 1997, losses follow a lognormal distribution with parameters $\mu + \ln k$ and σ, where k is greater than 1.
- In 1996, $100p\%$ of the losses exceed the mean of the losses in 1997.

Determine σ.

Note: z_p is the $100p^{\text{th}}$ percentile of a normal distribution with mean 0 and variance 1.

(A) $2\ln k$

(B) $-z_p \pm \sqrt{z_p^2 - 2\ln k}$

(C) $z_p \pm \sqrt{z_p^2 - 2\ln k}$

(D) $\sqrt{-z_p \pm \sqrt{z_p^2 - 2\ln k}}$

(E) $\sqrt{z_p \pm \sqrt{z_p^2 - 2\ln k}}$

2.6. **[4B-S94:16]** (1 point) You are given the following:

- Losses in 1993 follow the density function
$$f(x) = 3x^{-4}, \quad x \geq 1,$$
where x = losses in millions of dollars.
- Inflation of 10% impacts all claims uniformly from 1993 to 1994.

Determine the probability that losses in 1994 exceed 2.2 million.

(A) Less than 0.05

(B) At least 0.05, but less than 0.10

(C) At least 0.10, but less than 0.15

(D) At least 0.15, but less than 0.20

(E) At least 0.20

2.7. **[4B-F95:6]** (2 points) You are given the following:

- In 1994, losses follow a Pareto distribution with parameters $\theta = 500$ and $\alpha = 1.5$.
- Inflation of 5% impacts all losses uniformly from 1994 to 1995.

What is the median of the portion of the 1995 loss distribution above 200?

(A) Less than 600

(B) At least 600, but less than 620

(C) At least 620, but less than 640

(D) At least 640, but less than 660

(E) At least 660

Exercises continue on the next page . . .

2.8. **[CAS3-S04:34]** Claim severities are modeled using a continuous distribution and inflation impacts claims uniformly at an annual rate of i.

Which of the following are true statements regarding the distribution of claim severities after the effect of inflation?

1. An Exponential distribution will have scale parameter $(1+i)\theta$
2. A 2-parameter Pareto distribution will have scale parameters $(1+i)\alpha$ and $(1+i)\theta$.
3. A Paralogistic distribution will have scale parameter $\theta/(1+i)$

(A) 1 only (B) 3 only (C) 1 and 2 only (D) 2 and 3 only (E) 1, 2, and 3

2.9. **[CAS3-F05:21]** Losses during the current year follow a Pareto distribution with $\alpha = 2$ and $\theta = 400{,}000$. Annual inflation is 10%.

Calculate the ratio of the expected proportion of claims that will exceed \$750,000 next year to the proportion of claims that exceed \$750,000 this year.

(A) Less than 1.105
(B) At least 1.105, but less than 1.115
(C) At least 1.115, but less than 1.125
(D) At least 1.125, but less than 1.135
(E) At least 1.135

2.10. **[4B-S99:17]** You are given the following:

- In 1998, claim sizes follow a Pareto distribution with parameters θ (unknown) and $\alpha = 2$.
- Inflation of 6% affects all claims uniformly from 1998 to 1999.
- r is the ratio of the proportion of claims that exceed d in 1999 to the proportion of claims that exceed d in 1998.

Determine the limit of r as d goes to infinity.

(A) Less than 1.05
(B) At least 1.05, but less than 1.10
(C) At least 1.10, but less than 1.15
(D) At least 1.15, but less than 1.20
(E) At least 1.20

Exercises continue on the next page . . .

2.11. **[4B-F94:28]** (2 points) You are given the following:

- In 1993, the claim amounts for a certain line of business were normally distributed with mean $\mu = 1000$ and variance $\sigma^2 = 10{,}000$;

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) \quad -\infty < x < \infty, \quad \mu = 1000, \ \sigma = 100.$$

- Inflation of 5% impacted all claims uniformly from 1993 to 1994.

What is the distribution for claim amounts in 1994?

(A) No longer a normal distribution
(B) Normal with $\mu = 1000$ and $\sigma = 102.5$.
(C) Normal with $\mu = 1000$ and $\sigma = 105.0$.
(D) Normal with $\mu = 1050$ and $\sigma = 102.5$.
(E) Normal with $\mu = 1050$ and $\sigma = 105.0$.

2.12. **[4B-S93:11]** (1 point) You are given the following:

(i) The underlying distribution for 1992 losses is given by a lognormal distribution with parameters $\mu = 17.953$ and $\sigma = 1.6028$.
(ii) Inflation of 10% impacts all claims uniformly the next year.

What is the underlying loss distribution after one year of inflation?

(A) Lognormal with $\mu' = 19.748$ and $\sigma' = 1.6028$.
(B) Lognormal with $\mu' = 18.048$ and $\sigma' = 1.6028$.
(C) Lognormal with $\mu' = 17.953$ and $\sigma' = 1.7631$.
(D) Lognormal with $\mu' = 17.953$ and $\sigma' = 1.4571$.
(E) No longer a lognormal distribution

2.13. X follows an exponential distribution with mean 10.

Determine the mean of X^4.

2.14. You are given

(i) X is exponential with mean 2.
(ii) $Y = X^{1.5}$.

Calculate $\mathbf{E}[Y^2]$.

2.15. X follows a gamma distribution with parameters $\alpha = 2.5$ and $\theta = 10$.

$Y = 1/X$.

Evaluate $\mathrm{Var}(Y)$.

Solutions

2.1. Looking up the tables for the inverse gamma distribution, we see that the mode is $\frac{\theta}{\alpha+1}$ and the mean is $\frac{\theta}{\alpha-1}$, so

$$\frac{\theta}{\alpha+1} = 4$$

$$\frac{\theta}{\alpha-1}=6$$

Dividing the second line into the first,

$$\frac{\alpha-1}{\alpha+1}=\frac{4}{6}$$
$$\alpha=5 \qquad \theta=24$$

Then (γ_1 is the coefficient of skewness).

$$\begin{aligned}
\mathbf{E}\left[X^2\right] &= \frac{\theta^2}{(\alpha-1)(\alpha-2)} = \frac{24^2}{12} = 48\\
\operatorname{Var}(X) &= 48-6^2 = 12\\
\mathbf{E}\left[X^3\right] &= \frac{\theta^3}{(\alpha-1)(\alpha-2)(\alpha-3)} = \frac{24^3}{24} = 576\\
\gamma_1 &= \frac{576-3(48)(6)+2(6^3)}{12^{1.5}}\\
&= \frac{144}{12^{1.5}} = \sqrt{12} = \boxed{\mathbf{3.4641}} \qquad \textbf{(E)}
\end{aligned}$$

2.2. The mean in 2021 is $\theta/3$. By definition, the 65th percentile is the number π_{65} such that $F(\pi_{65})=0.65$, so $F(\theta/3)=0.65$ for the 2022version of F. In 2022, F is two-parameter Pareto with inflated parameter $\theta'=(1+r)\theta$ and $\alpha=4$, so

$$\begin{aligned}
1-\left(\frac{\theta'}{\theta'+(\theta/3)}\right)^4 &= 0.65\\
\frac{(1+r)\theta}{(1+r)\theta+\theta/3} &= \sqrt[4]{0.35}\\
\frac{1+r}{4/3+r} &= \sqrt[4]{0.35}\\
r(1-\sqrt[4]{0.35}) &= \frac{4}{3}\sqrt[4]{0.35}-1\\
r &= \frac{(4/3)\sqrt[4]{0.35}-1}{1-\sqrt[4]{0.35}} = \boxed{\mathbf{0.1107}}
\end{aligned}$$

2.3. To scale a lognormal, you leave σ and add the logarithm of the scale to μ. So already, B and D are the only possibilities, and the question is whether you add or subtract $\ln 1.3$. But clearly the mean in dollars is higher than the mean in euros, so you add $\ln 1.3$ and get **(D)**.

2.4. Let X be the original variable, $Z=1.2X$. Since the mean is 25,000, the parameter θ is $25{,}000(\alpha-1)=50{,}000$.

$$\begin{aligned}
\Pr(X>100{,}000) &= \left(\frac{50}{150}\right)^3 = \frac{1}{27}\\
\Pr(Z>100{,}000) &= \left(\frac{60}{160}\right)^3 = \frac{27}{512}\\
\frac{27}{512}-\frac{1}{27} &= \boxed{\mathbf{0.0157}} \qquad \textbf{(A)}
\end{aligned}$$

2.5. The key is to understand (iii). If (for example) 30% of losses exceed \$10000, what percentage does not exceed \$10000? (Answer: 70%) And what percentile of the distribution of losses is \$10000? (Answer: 70^{th}). So statement (iii) is saying that the $100(1-p)^{\text{th}}$ percentile of losses in 1996 equals the mean of losses in 1997. Got it?

The mean of 1997 losses is $\exp(\mu + \ln k + \frac{\sigma^2}{2})$. The $100(1-p)$th percentile is $\exp(\mu - z_p\sigma)$. So:

$$\mu - z_p\sigma = \mu + \ln k + \frac{\sigma^2}{2}$$

$$\frac{\sigma^2}{2} + \sigma z_p + \ln k = 0$$

$$\sigma = \boxed{-z_p \pm \sqrt{z_p^2 - 2\ln k}} \qquad \textbf{(B)}$$

Notice that p must be less than 0.5, by the following reasoning. In general, the median of a lognormal (e^{μ}) is less than (or equal to, if $\sigma = 0$) the mean ($e^{\mu+\sigma^2/2}$), so the median of losses in 1996 is no more than the mean of losses in 1996, which in turn is less than the mean of losses in 1997 since $k > 1$, so $100p$ must be less than 50. Since p is less than 0.5, it follows that z_p will be negative, and σ is therefore positive, as it should be.

2.6. We recognize the 1993 distribution as a single-parameter Pareto with $\theta = 1$, $\alpha = 3$. The inflated parameters are $\theta = 1.1$, $\alpha = 3$. $\left(\frac{1.1}{2.2}\right)^3 = \boxed{0.125}$. **(C)**

2.7. Let X be the inflated variable, with $\theta = 525$, $\alpha = 1.5$. $\Pr(X > 200) = \left(\frac{525}{525+200}\right)^{1.5} = 0.6162$. Let F be the original distribution function, F^* the distribution of $X \mid X > 200$. Then $F(200) = 1 - 0.6162 = 0.3838$ and

$$F^*(x) = \Pr(X \le x \mid X > 200) = \frac{\Pr(200 < X \le x)}{\Pr(X > 200)} = \frac{F(x) - F(200)}{1 - F(200)}$$

So to calculate the median, we set $F^*(x) = 0.5$, which means

$$\frac{F(x) - F(200)}{1 - F(200)} = 0.5$$

$$\frac{F(x) - 0.3838}{0.6162} = 0.5$$

$$F(x) = 0.5(0.6162) + 0.3838 = 0.6919$$

We must find x such that $F(x) = 0.6919$.

$$1 - \left(\frac{525}{525 + x}\right)^{1.5} = 0.6919$$

$$\frac{525}{525 + x} = 0.4562$$

$$\frac{525 - 525(0.4562)}{0.4562} = x$$

$$x = \boxed{625.87} \qquad \textbf{(C)}$$

2.8. All the distributions are parameterized so that θ is the scale parameter and is multiplied by $1 + i$; no other parameters change, and you should never divide by $1 + i$. Therefore **only 1** is correct. **(A)**

2.9. The Pareto is a scale distribution with scale parameter θ, so annual inflation of 10% increases θ by 10%, making it 440,000. The proportion of claims above 750,000 is $S(750{,}000) = \left(\frac{\theta}{\theta+750{,}000}\right)^{\alpha}$. Hence, the proportion this year is $\left(\frac{400{,}000}{400{,}000+750{,}000}\right)^2 = 0.120983$ and the proportion next year is $\left(\frac{440{,}000}{440{,}000+750{,}000}\right)^2 = 0.136714$. The ratio is $\frac{0.136714}{0.120983} = \boxed{1.1300}$. **(D)**

2.10. This is:

$$\frac{\left(\frac{1.06\theta}{1.06\theta+d}\right)^2}{\left(\frac{\theta}{\theta+d}\right)^2} = \frac{1.06^2(\theta+d)^2}{(1.06\theta+d)^2} \longrightarrow 1.06^2 = \boxed{1.1236}. \qquad \textbf{(C)}$$

2.11. If X is normal, then $aX+b$ is normal as well. In particular, $1.05X$ is normal. So the distribution of claims after 5% uniform inflation is normal.

For any distribution, multiplying the distribution by a constant multiplies the mean and standard deviation by that same constant. Thus in this case, the new mean is 1050 and the new standard deviation is 105. **(E)**

2.12. Add $\ln 1.1$ to μ: $17.953 + \ln 1.1 = 18.048$. σ does not change. **(B)**

2.13. The k^{th} moment for an exponential is given in the tables:

$$\mathbf{E}[X^k] = k!\theta^k$$

for $k = 4$ and the mean $\theta = 10$, this is $4!(10^4) = \boxed{240{,}000}$.

2.14. While Y is Weibull, you don't need to know that. It's simpler to use $Y^2 = X^3$ and look up the third moment of an exponential.

$$\mathbf{E}[X^3] = 3!\theta^3 = 6(2^3) = \boxed{48}$$

2.15. We calculate $\mathbf{E}[Y]$ and $\mathbf{E}[Y^2]$, or $\mathbf{E}[X^{-1}]$ and $\mathbf{E}[X^{-2}]$. Note that the special formula in the tables for integral moments of a gamma, $\mathbf{E}[X^k] = \theta^k(\alpha+k-1)\cdots\alpha$ only applies when k is a *positive* integer, so it cannot be used for the -1 and -2 moments. Instead, we must use the general formula for moments given in the tables,

$$\mathbf{E}[X^k] = \frac{\theta^k\Gamma(\alpha+k)}{\Gamma(\alpha)}$$

For $k=-1$, this is

$$\mathbf{E}[X^{-1}] = \frac{\theta^{-1}\Gamma(\alpha-1)}{\Gamma(\alpha)} = \frac{1}{\theta(\alpha-1)}$$

since $\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$. For $k=-2$,

$$\mathbf{E}[X^{-2}] = \frac{1}{\theta^2(\alpha-1)(\alpha-2)}$$

Therefore,

$$\operatorname{Var}(Y) = \frac{1}{10^2(1.5)(0.5)} - \left(\frac{1}{10(1.5)}\right)^2 = \boxed{0.00888889}$$

Part VII

Practice Exams

It's now time to see how well you would do on an exam.

The distribution of topics on these 12 exams is approximately the distribution specified by the syllabus. The difficulty of the exams is random; all exams have questions of varying levels of difficulty.

Practice Exam 1

1. For a health insurance coverage, there are two types of policyholders.

75% of policyholders are healthy. Annual claim costs for those policyholders have mean 2,000 and variance 10,000,000.

25% of policyholders are in bad health. Annual claim costs for those policyholders have mean 10,000 and variance 50,000,000.

Calculate the variance of annual claim costs for a policyholder selected at random.

(A) 20,000,000 (B) 28,000,000 (C) 32,000,000 (D) 44,000,000 (E) 48,000,000

2. Bill Driver and Jane Motorist are involved in an automobile accident. Jane Motorist's car is totally destroyed. Its value before the accident was 8000, and the scrap metal after the accident is worth 500. Bill Driver is at fault.

Big Insurance Company insures Jane Motorist. Jane has liability insurance with a 100,000 limit and collision insurance with a 1000 deductible.

Standard Insurance Company insures Bill Driver. Bill has liability insurance with a 50,000 limit and collision insurance with a 500 deductible.

Jane files a claim with Big Insurance Company and receives 7000.

Calculate the net amount that Big Insurance Company receives (net of payment of subrogation proceeds to Jane) from subrogation.

(A) 6000 (B) 6500 (C) 7000 (D) 7500 (E) 8000

3. An excess of loss catastrophe reinsurance treaty covers the following layers, expressed in millions:

80% of 100 excess of 100
85% of 200 excess of 200
90% of 400 excess of 400

Calculate the reinsurance payment for a catastrophic loss of 650 million.

(A) 225 million (B) 475 million (C) 495 million (D) 553 million (E) 585 million

4. A rate filing for six-month policies will be effective starting October 1, CY6 for 2 years.

Losses for this rate filing were incurred in AY1 and the amount paid through 12/31/AY4 is 3,500,000.

Trend is at annual effective rate of 6.5%.

Loss development factors are:

$$3/2\text{: } 1.50, \qquad 4/3\text{: } 1.05, \qquad \infty/4\text{: } 1.05$$

Calculate trended and developed losses for AY1.

(A) Less than 5,600,000
(B) At least 5,600,000, but less than 5,700,000
(C) At least 5,700,000, but less than 5,800,000
(D) At least 5,800,000, but less than 5,900,000
(E) At least 5,900,000

5. You are given

Accident Year	Cumulative Payments through Development Year 0	1	2	Earned Premium
AY1	25,000	41,000	48,000	120,000
AY2	30,000	45,000		140,000
AY3	33,000			150,000

The loss ratio is 60%.

Calculate the loss reserve using the loss ratio method.

(A) 100,000 (B) 105,000 (C) 110,000 (D) 115,000 (E) 120,000

6. You are given the following observations:

2, 10, 28, 64, 100

The observations are fitted to an inverse exponential distribution using maximum likelihood.

Determine the resulting estimate of the mode.

(A) 3.2 (B) 3.4 (C) 3.6 (D) 3.8 (E) 4.0

7. You own 100 shares of a stock whose current price is 42. You would like to hedge your downside exposure by buying 100 6-month European put options with a strike price of 40. You are given:

(i) The Black-Scholes-Merton framework is assumed.
(ii) The continuously compounded risk-free interest rate is 5%.
(iii) The stock pays no dividends.
(iv) The stock follows a lognormal process with $\mu = 0.06$ and $\sigma = 0.22$.

Determine the cost of the put options.

(A) 121 (B) 123 (C) 125 (D) 127 (E) 129

Exam questions continue on the next page . . .

8. At a company, the number of sick days taken by each employee in a year follows a Poisson distribution with mean λ. Over all employees, the distribution of λ has the following density function:

$$f(\lambda) = \frac{\lambda e^{-\lambda/3}}{9}$$

Calculate the probability that an employee selected at random will take more than 2 sick days in a year.

(A) 0.59 (B) 0.62 (C) 0.66 (D) 0.70 (E) 0.74

9. For loss size X, you are given:

x	$\mathbf{E}[X \wedge x]$
1000	400
2000	700
3000	900
4000	1000
5000	1100
∞	2500

An insurance coverage has an ordinary deductible of 2000.

Calculate the loss elimination ratio after 100% inflation if the deductible is not changed.

(A) 0.08 (B) 0.14 (C) 0.16 (D) 0.20 (E) 0.40

10. In a study on loss sizes on automobile liability coverage, you are given:

(i) 5 observations $x_1, \ldots, x_5$ from a plan with no deductible and a policy limit of 10,000.
(ii) 5 observations at the limit from a plan with no deductible and a policy limit of 10,000.
(iii) 5 observations $y_1, \ldots, y_5$ from a plan with a deductible of 10,000 and no policy limit.

Which of the following is the likelihood function for this set of observations?

(A) $\prod_{i=1}^{5} f(x_i) \prod_{i-1}^{5} f(y_i)$

(B) $\bigl(1 - F(10{,}000)\bigr)^5 \prod_{i=1}^{5} f(x_i) \prod_{i-1}^{5} f(y_i)$

(C) $\dfrac{\bigl(1 - F(10{,}000)\bigr)^5 \prod_{i=1}^{5} f(x_i) \prod_{i-1}^{5} f(y_i)}{\bigl(F(10{,}000)\bigr)^5}$

(D) $\dfrac{\prod_{i=1}^{5} f(x_i) \prod_{i=1}^{5} f(y_i)}{\bigl(1 - F(10{,}000)\bigr)^5}$

(E) $\dfrac{\bigl(F(10{,}000)\bigr)^5 \prod_{i=1}^{5} f(x_i) \prod_{i-1}^{5} f(y_i)}{\bigl(1 - F(10{,}000)\bigr)^5}$

Exam questions continue on the next page . . .

11. A study on claim sizes produced the following results:

Claim size	Number	Deductible	Limit
500	4	None	10,000
1000	4	500	None
2000	3	500	None
5000	2	None	10,000
At limit	5	None	10,000

A single-parameter Pareto with $\theta = 400$ is fitted to the data using maximum likelihood.

Determine the estimate of α.

(A) 0.43 (B) 0.44 (C) 0.45 (D) 0.61 (E) 0.62

12. You are given:

Accident Year	Incurred Losses through Development Year 0	1	2	3	4	Earned Premium
AY1	7,800	8,900	9,500	11,000	11,000	16,000
AY2	9,100	9,800	10,500	10,800		20,000
AY3	8,600	9,500	10,100			23,000
AY4	9,500	10,000				24,000
AY5	10,700					25,000

The expected loss ratio is 0.7.

Losses mature at the end of 3 years.

Calculate the IBNR reserve using the Bornhuetter-Ferguson method with volume-weighted average loss development factors.

(A) 7,100 (B) 7,200 (C) 7,300 (D) 7,400 (E) 7,500

13. On an automobile liability coverage, annual claim counts follow a negative binomial distribution with mean 0.2 and variance 0.3. Claim sizes follow a two-parameter Pareto distribution with $\alpha = 3$ and $\theta = 10$. Claim counts and claim sizes are independent.

Calculate the variance of annual aggregate claim costs.

(A) 22.5 (B) 25.0 (C) 27.5 (D) 32.5 (E) 35.0

14. A minor medical insurance coverage has the following provisions:

(i) Annual losses in excess of 10,000 are not covered by the insurance.
(ii) The policyholder pays the first 1,000 of annual losses.
(iii) The insurance company pays 60% of the excess of annual losses over 1,000, after taking into account the limitation mentioned in (i).

Annual losses follow a two-parameter Pareto distribution with $\alpha = 4$ and $\theta = 8000$.

Calculate expected annual payments for one policyholder under this insurance.

(A) 983 (B) 1004 (C) 1025 (D) 1046 (E) 1067

Exam questions continue on the next page . . .

15. A company has 100 shares of ABC stock. The current price of ABC stock is 30. ABC stock pays no dividends. The company would like to guarantee its ability to sell the stock at the end of six months for at least 28. European call options on the same stock expiring in 6 months with exercise price 28 are available for 4.10. The continuously compounded risk-free interest rate is 5%.

Determine the cost of the hedge.

(A) 73 (B) 85 (C) 99 (D) 126 (E) 141

16. Let X be the random variable with distribution function

$$F_X(x) = 1 - 0.6e^{-x/10} - 0.4e^{-x/20}$$

Calculate $\mathrm{TVaR}_{0.95}(X)$.

(A) 59 (B) 60 (C) 61 (D) 62 (E) 63

17. For a discrete probability distribution in the $(a, b, 0)$ class, you are given

(i) $p_2 = 0.0768$
(ii) $p_3 = p_4 = 0.08192$

Determine p_0.

(A) 0.02 (B) 0.03 (C) 0.04 (D) 0.05 (E) 0.06

18. Losses on an insurance coverage follow a distribution with density function

$$f(x) = \frac{3}{100^3}(100 - x)^2 \qquad 0 \le x \le 100$$

Losses are subject to an ordinary deductible of 15.

Calculate the loss elimination ratio.

(A) 0.46 (B) 0.48 (C) 0.50 (D) 0.52 (E) 0.54

19. A reinsurance company offers a stop-loss reinsurance contract that pays the excess of annual aggregate losses over 3.

You are given:

(i) Loss counts follow a binomial distribution with $m = 3$ and $q = 0.2$.
(ii) Loss sizes have the following distribution:

Size	Probability
1	0.6
2	0.2
3	0.1
4	0.1

Calculate the expected annual payment under the stop-loss reinsurance contract.

(A) 0.09 (B) 0.10 (C) 0.11 (D) 0.12 (E) 0.13

Exam questions continue on the next page . . .

20. Annual claim frequency follows a Poisson distribution. Loss sizes follow a Weibull distribution with $\tau = 0.5$.

Full credibility for aggregate loss experience is granted if the probability that aggregate losses differ from expected by less than 6% is 95%.

Determine the number of expected claims needed for full credibility.

(A) 6403 (B) 6755 (C) 7102 (D) 7470 (E) 7808

Solutions to the above questions begin on page 627.

Auto-Grade My Exam (Click Here)

Practice Exam 1

1. **[Section 4.1]** You may either use the conditional variance formula (equation (4.1)), or compute first and second moments.

With the conditional variance formula, let I be the indicator of whether the policyholder is healthy or in bad health. Let X be annual claim counts then

$$\begin{aligned}\mathrm{Var}(X) &= \mathrm{Var}_I\big(\mathbf{E}_X[X \mid I]\big) + \mathbf{E}_I\big[\mathrm{Var}_X(X \mid I)\big]\\ &= \mathrm{Var}_I(2000, 10{,}000) + \mathbf{E}_I[10{,}000{,}000, 50{,}000{,}000]\end{aligned}$$

where $\mathrm{Var}_I(2{,}000, 10{,}000)$ means the variance of a random variable that is 2,000 with probability 0.75 and 10,000 with probability 0.25. By the Bernoulli shortcut, the variance of such a random variable is

$$(0.75)(0.25)(10{,}000 - 2{,}000)^2 = 12{,}000{,}000$$

$\mathbf{E}_I[10{,}000{,}000, 50{,}000{,}000]$ means the expected value of a random variable that is 10,000,000 with probability 0.75 and 50,000,000 with probability 0.25. The expected value of such a random variable is

$$0.75(10{,}000{,}000) + 0.25(50{,}000{,}000) = 20{,}000{,}000$$

Adding up the variance of the mean and the mean of the variances, we get $\mathrm{Var}(X) = 12{,}000{,}000 + 20{,}000{,}000 =$ **32,000,000**. **(C)**

With first and second moments, the overall first moment of annual claim counts is

$$0.75(2{,}000) + 0.25(10{,}000) = 4{,}000$$

The overall second moment of annual claim counts is the weighted average of the individual second moments, and the second moment for each type of driver is the variance plus the mean squared.

$$0.75(10{,}000{,}000 + 2{,}000^2) + 0.25(50{,}000{,}000 + 10{,}000^2) = 48{,}000{,}000$$

The overall variance of claim counts is $48{,}000{,}000 - 4{,}000^2 =$ **32,000,000**. **(C)**

2. **[Lesson 5]** Big Insurance Company pays Jane 7000 and receives the scrap metal, for a net loss of 6500. That is the amount that it gets upon subrogration. The remaining 1000 of the subrogation is paid to Jane. **(B)**

3. **[Lesson 13]** The layers are 100–200, 200–400, and 400–800. The amount of the catastrophic loss in each of those layers is 100, 200, and 250 respectively. The reinsurance pays $0.8(100) + 0.85(200) + 0.9(250) =$ **475 million**. **(B)**

4. **[Lesson 9]** Average date of sale of these policies is 10/1/CY7 and average date of accident is 3 months later, or 1/1/CY8. Trend is from 7/1/CY1 through 1/1/CY8, or 6.5 years. Paid data is for year 3, so future development is $(1.05)(1.05) = 1.1025$. Trended and developed losses are $3{,}500{,}000(1.1025)(1.065^{6.5}) =$ **5,810,575**. **(D)**

5. **[Section 7.2]** Projected losses based on the loss ratio are

$$0.6(120{,}000 + 140{,}000 + 150{,}000) = 246{,}000$$

Paid to date is $48{,}000 + 45{,}000 + 33{,}000 = 126{,}000$. The reserve is $246{,}000 - 126{,}000 =$ **120,000**. **(E)**

6. **[Lesson 23]** The likelihood function, ignoring the multiplicative constant $1/\prod x_i^2$, is

$$L(\theta) = \theta^5 e^{-\theta \sum 1/x_i}$$